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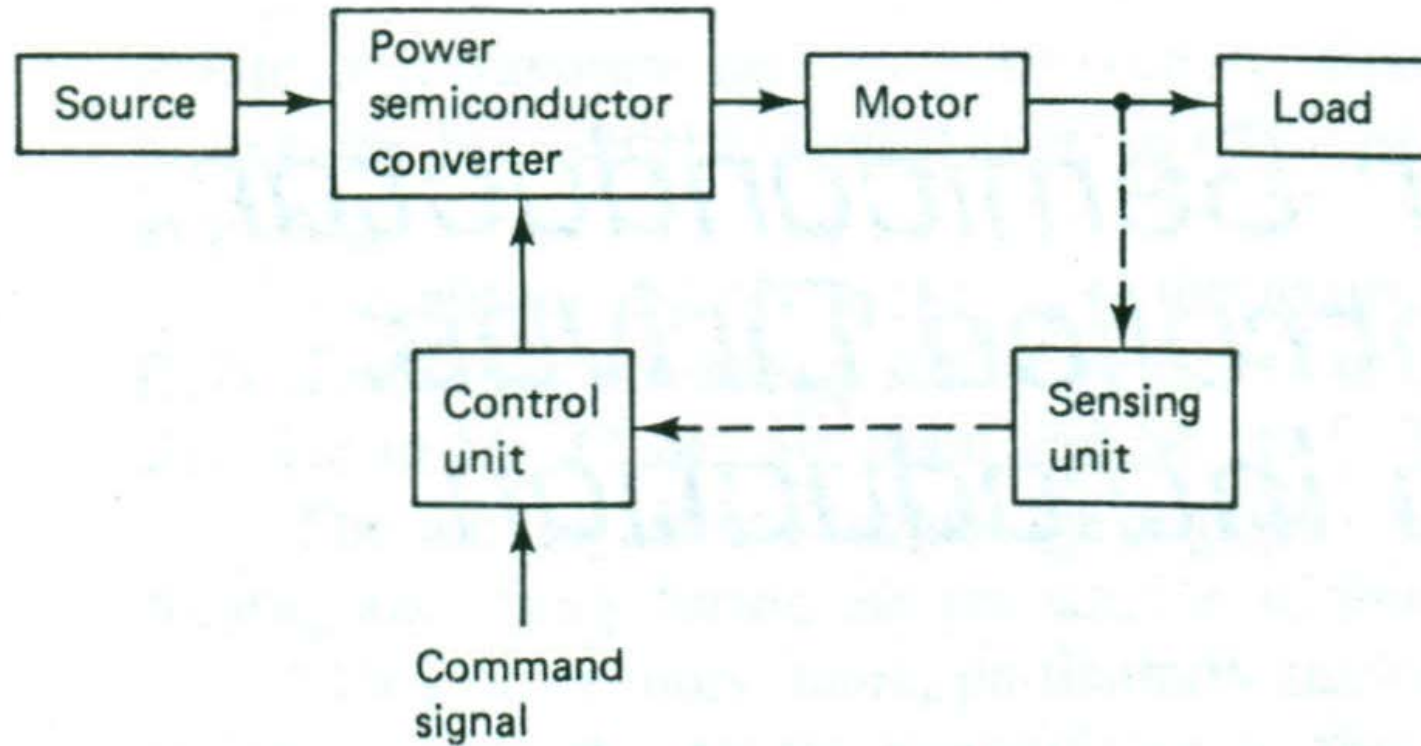
Electrical Machine Drive and Special Machines

ENEE5303

Introduction

- A modern variable speed system has four components:
 1. Power converters or drive circuits
 2. Electric machines
 3. Control system
 4. Load

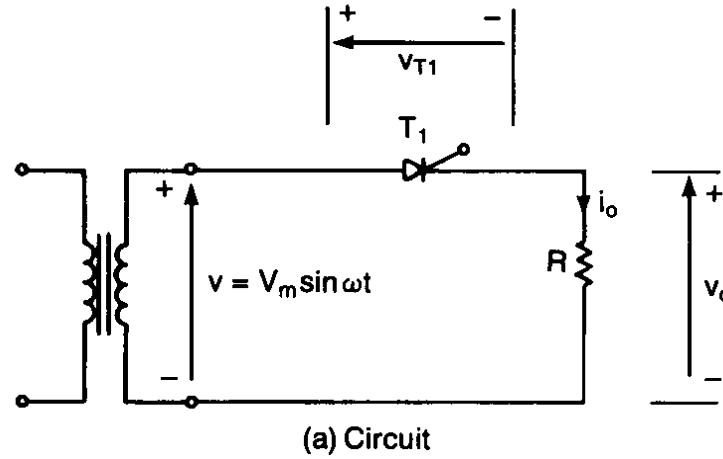
- A Block diagram of an electric drive



1. Power Converters

1.1 Controlled rectifiers

- Types of controlled rectifiers (single and three-phase)
 - Half-wave
 - Semiconverter
 - Full-wave
 - Dual
- They are fed from 1-phase or 3-phase AC main supply and provide a variable DC output voltage for control of DC motors.
- or sometimes input DC supply to the inverters in the case of AC machines

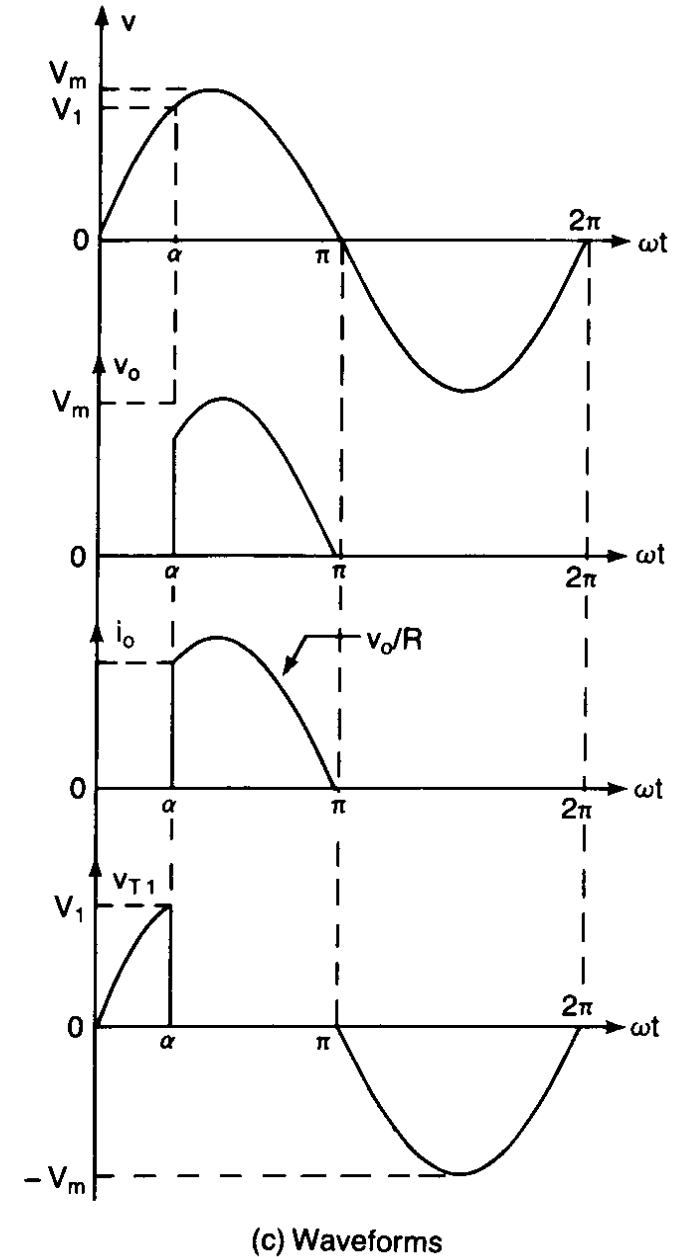
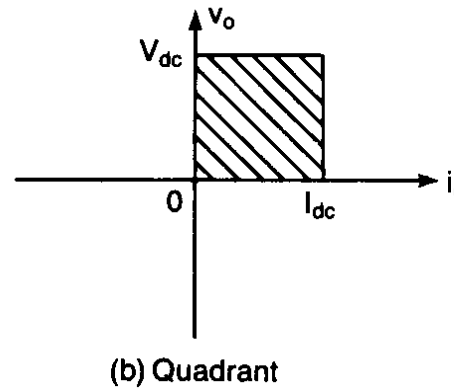


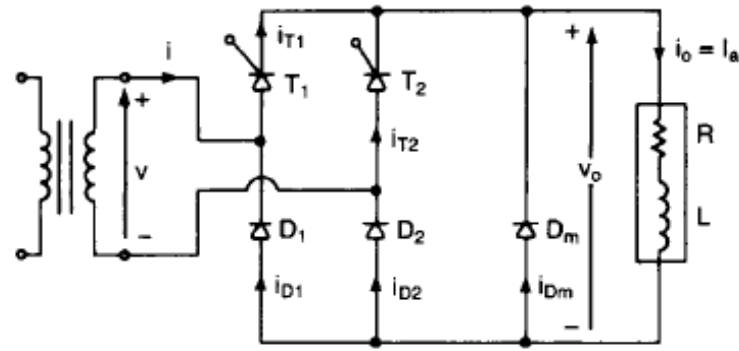
Single-phase:

Half-wave controlled rectifier

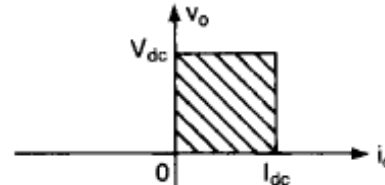
The average DC output voltage is given by:

$$V_{DC} = \frac{V_m}{2\pi} (1 + \cos \alpha)$$

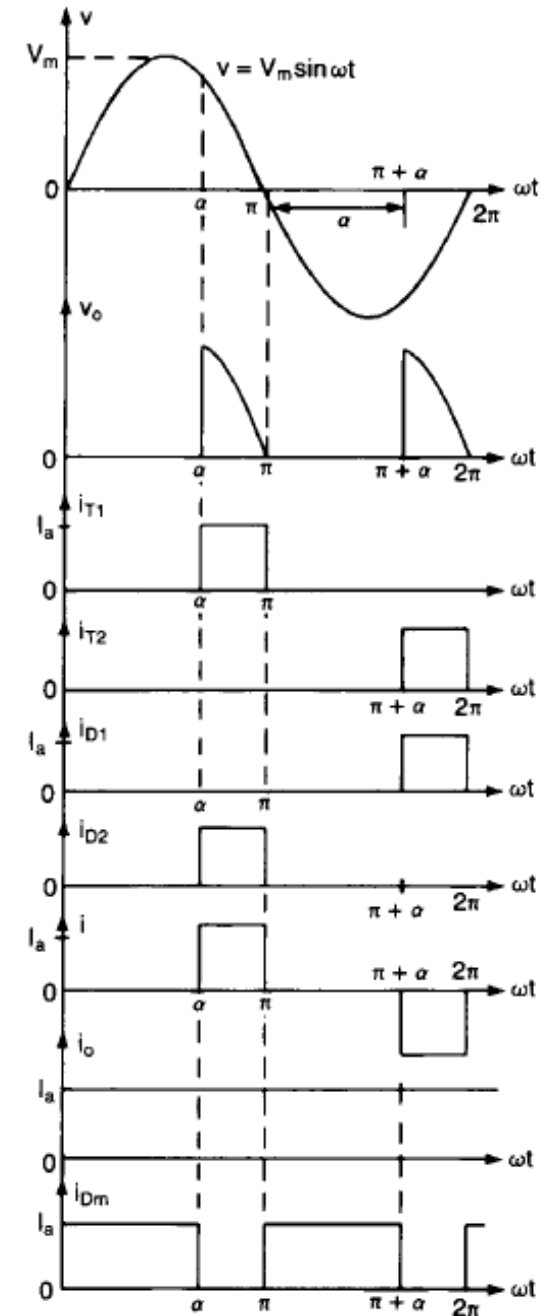




(a) Circuit



(b) Quadrant



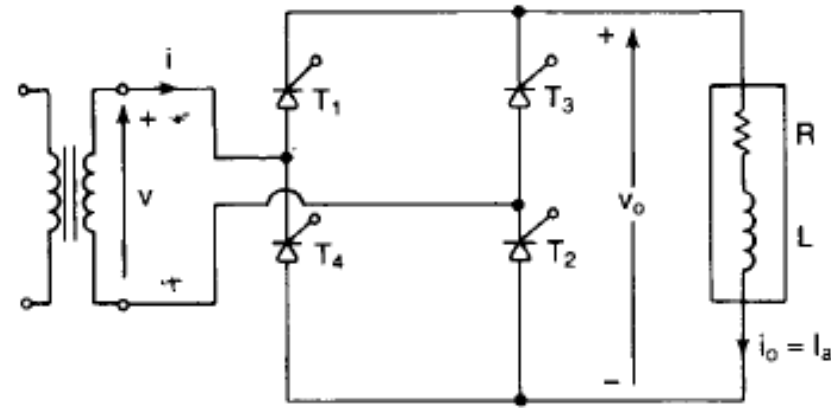
(c) Waveforms

Single-phase:

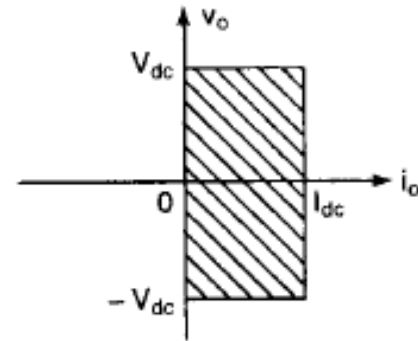
Semiconverter

The average DC output voltage is given by:

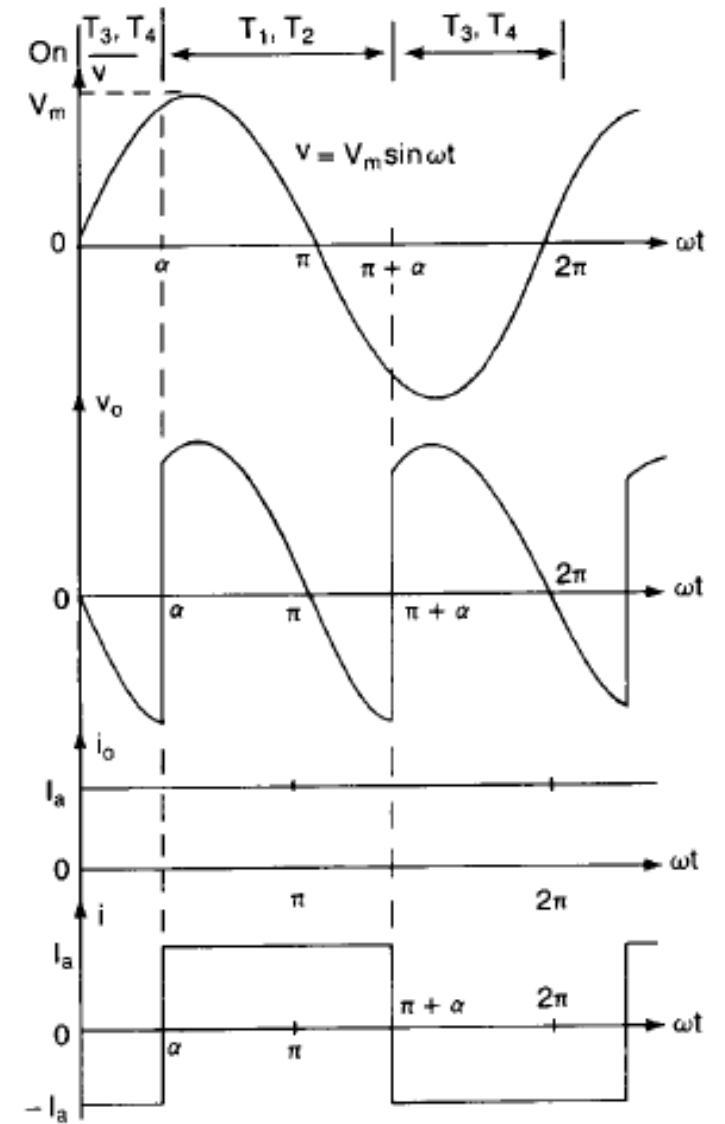
$$V_{DC} = \frac{V_m}{\pi} (1 + \cos \alpha)$$



(a) Circuit



(b) Quadrant



(c) Waveforms

Single-phase:

Full-wave controlled rectifier

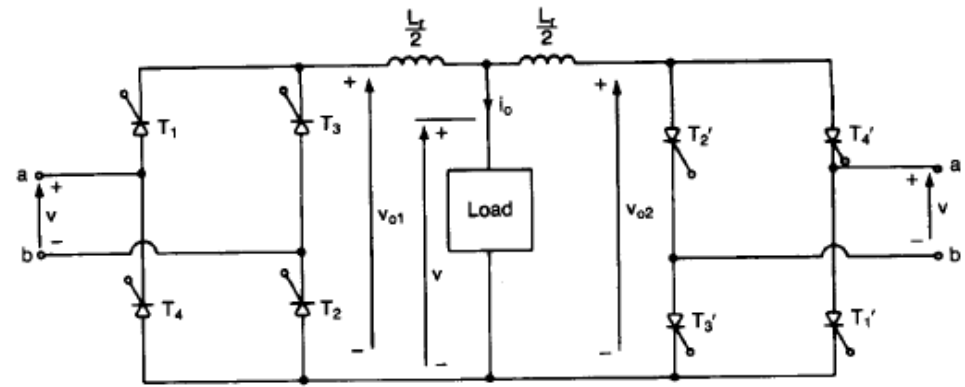
The average DC output voltage is given by:

$$V_{DC} = \frac{2V_m}{\pi} \cos \alpha$$

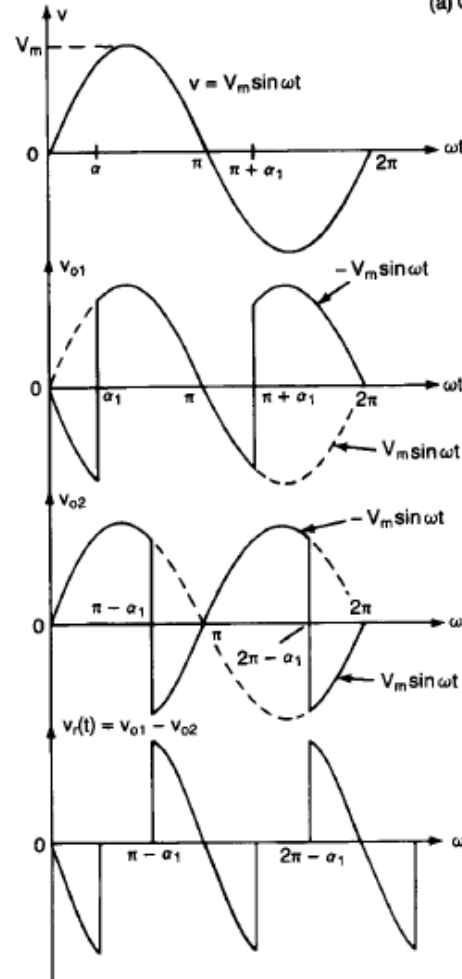
*Single-phase:
Dual converter*

The average DC output voltage is given by:

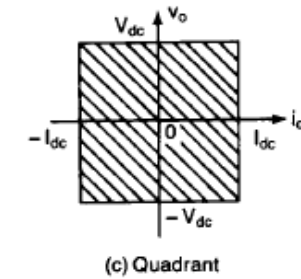
$$V_{DC} = \frac{2V_m}{\pi} \cos \alpha$$



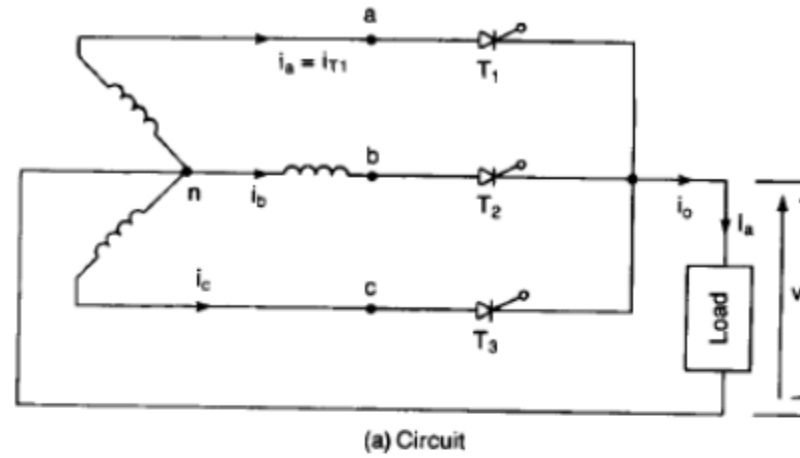
(a) Circuit



(b) Waveforms



(c) Quadrant

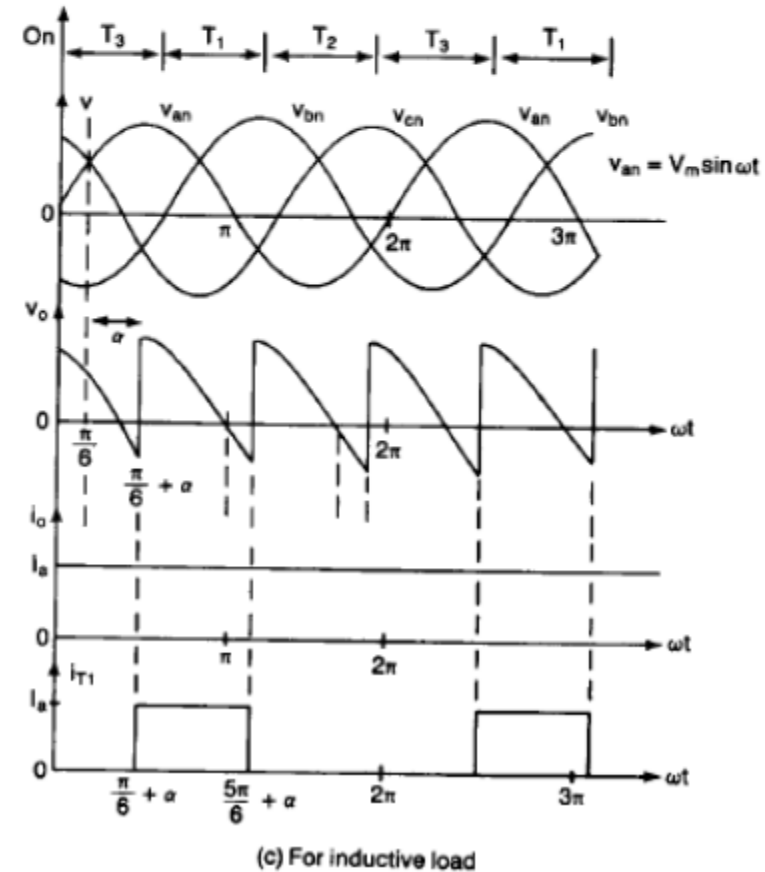


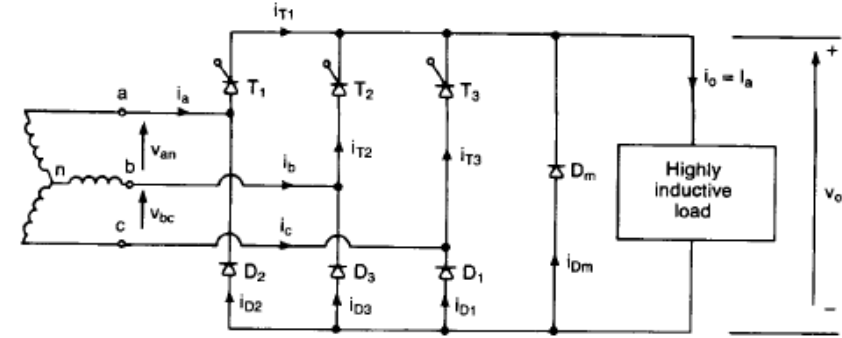
Three-phase:

Half-wave controlled rectifier

The average DC output voltage is given by:

$$V_{DC} = \frac{3\sqrt{3}}{2\pi} V_m \cos \alpha$$





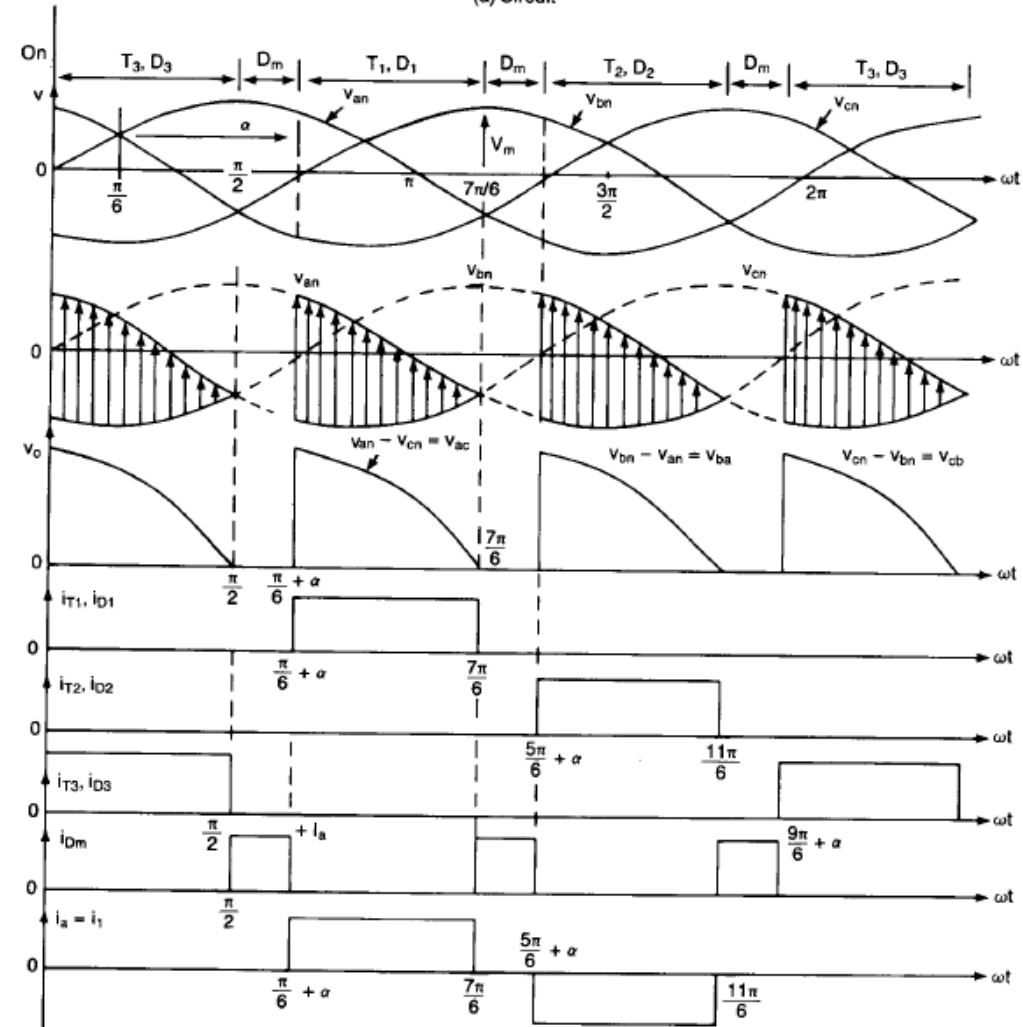
(a) Circuit

Three-phase:

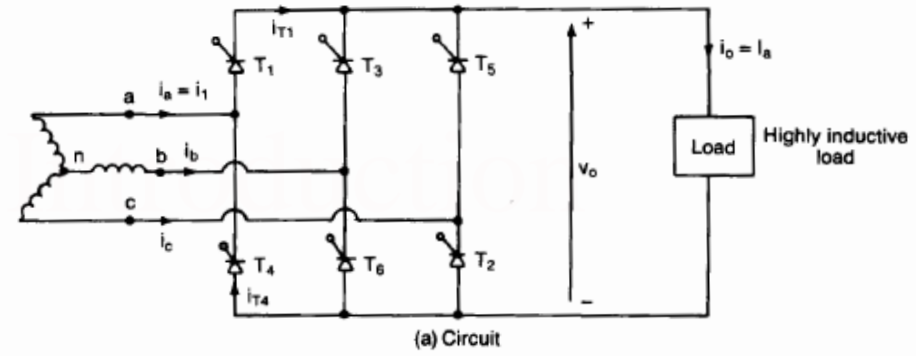
Semiconverter

The average DC output voltage is given by:

$$V_{DC} = \frac{3\sqrt{3}}{2\pi} V_m (1 + \cos \alpha)$$



(b) Waveforms for $\alpha = 90^\circ$

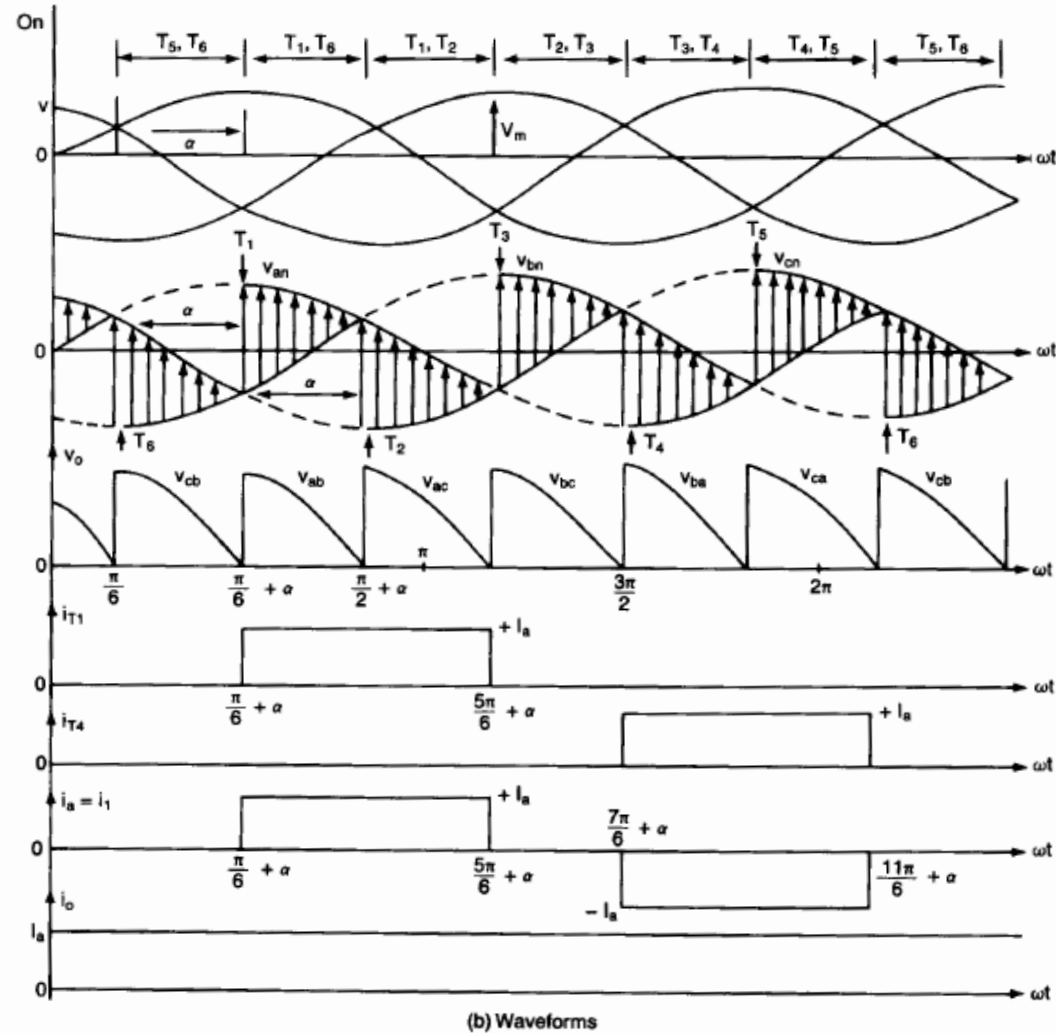


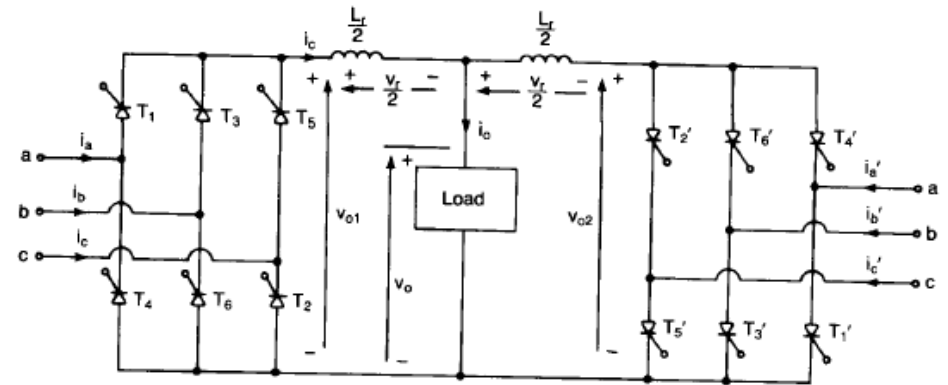
Three-phase:

Full-wave controlled rectifier

The average DC output voltage is given by:

$$V_{DC} = \frac{3\sqrt{3}}{\pi} V_m \cos \alpha$$





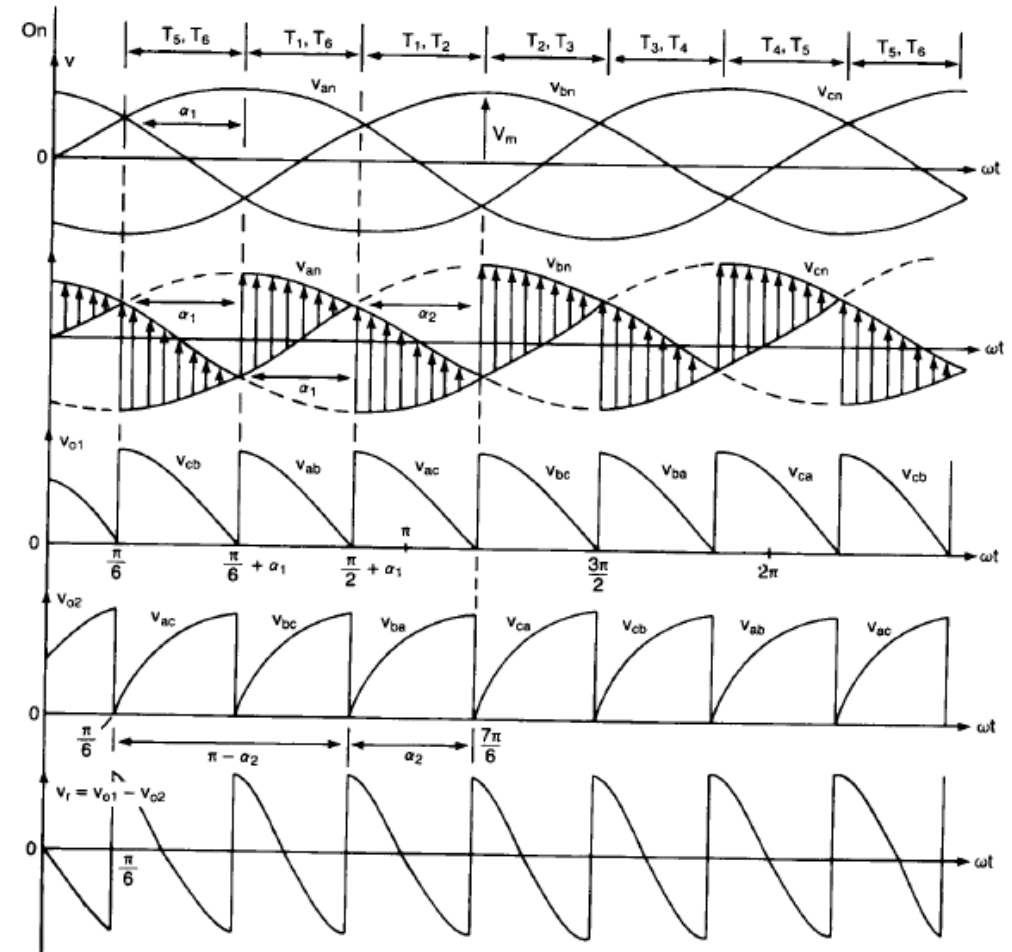
(a) Circuit

Three-phase:

Dual converter

The average DC output voltage is given by:

$$V_{DC} = \frac{3\sqrt{3}}{\pi} V_m \cos \alpha$$



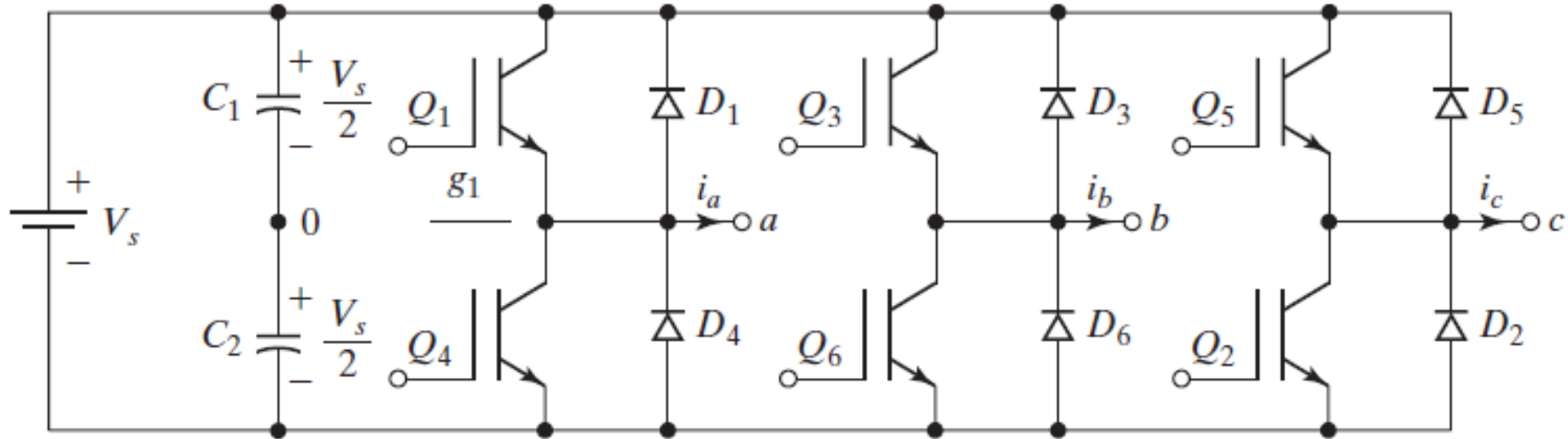
(b) Waveforms

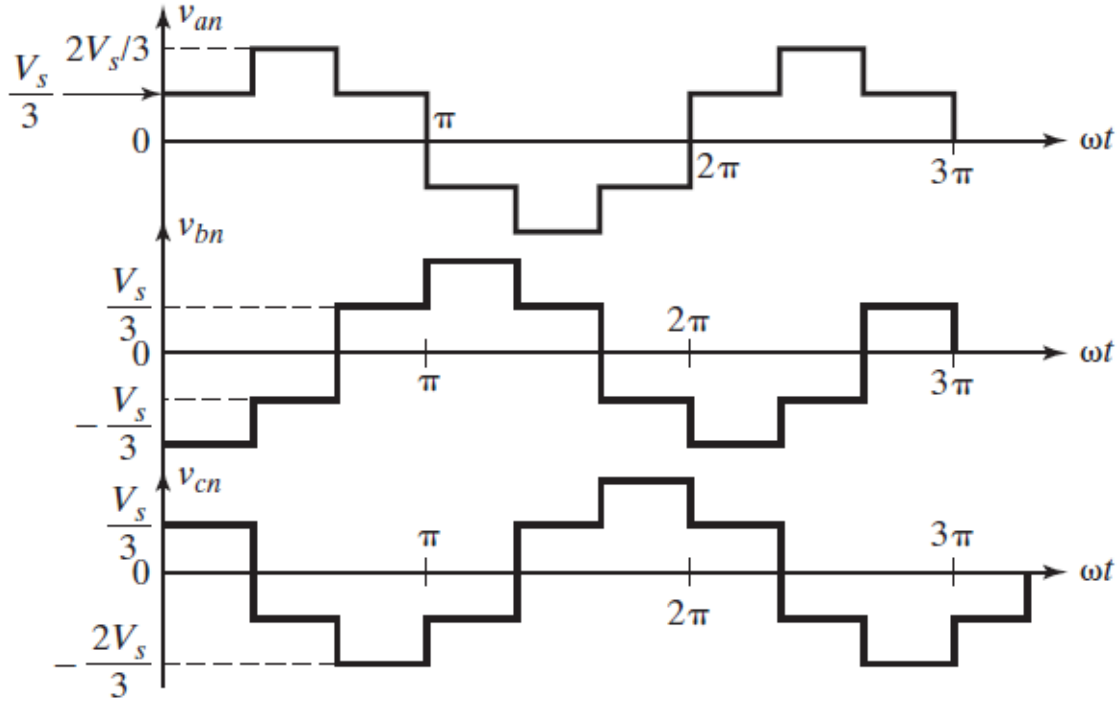
1.2 Inverters

- Types of inverters used for AC machine drive
 - Six-step inverters
 - Voltage source inverters (VSIs)
- They provide variable AC voltages and currents at desired frequency and phase for the drive of AC machines.
- The DC supply input to the inverter is derived either from a battery as in the case of electric vehicles or from a rectified AC source with controlled or uncontrolled rectifiers

- The DC supply input to the six-step inverter is derived from a rectified AC source with controlled rectifiers (or battery)
- The DC supply input to the VSI is derived from a rectified AC source with uncontrolled rectifiers (or battery)

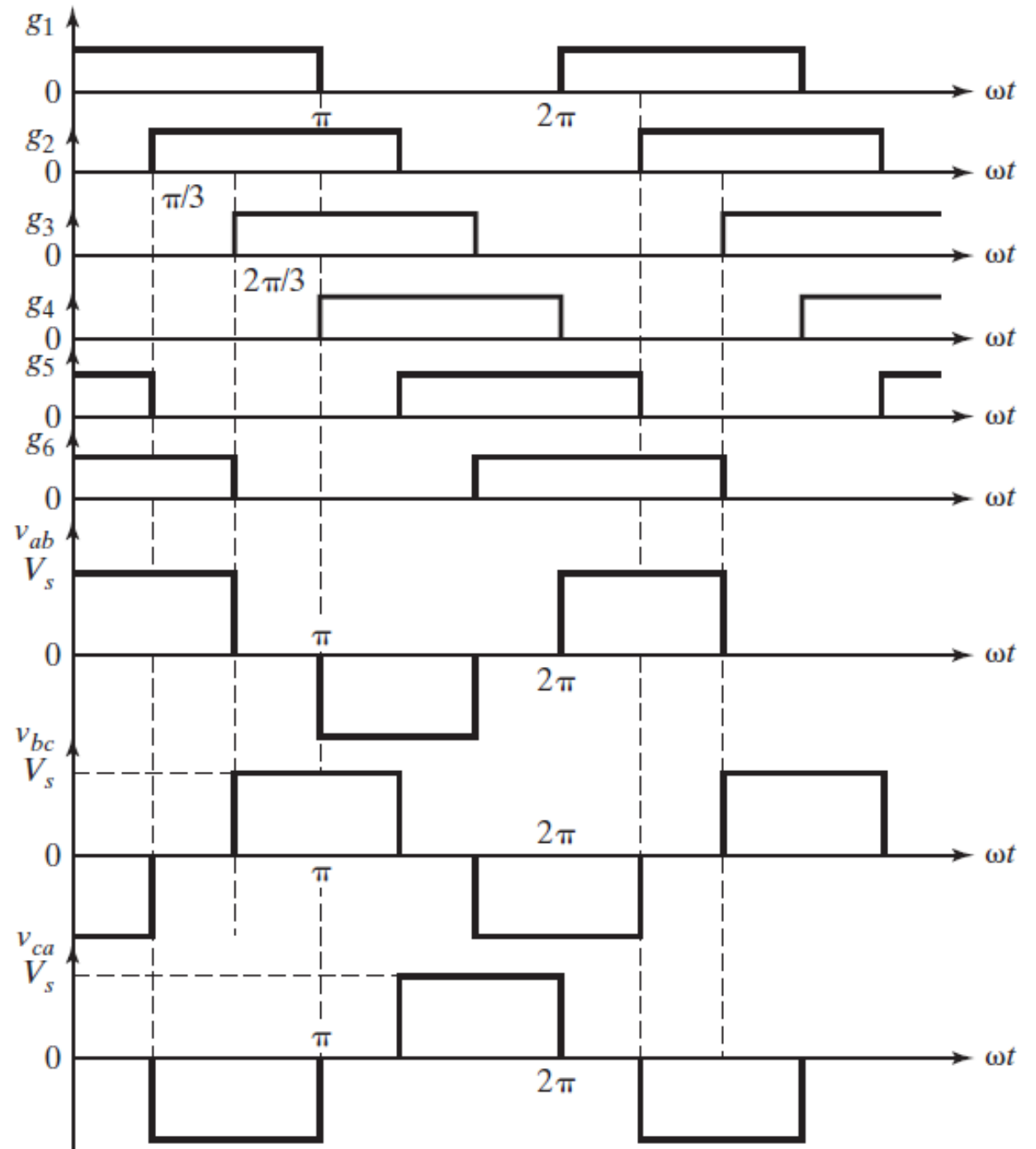
Six-step inverter



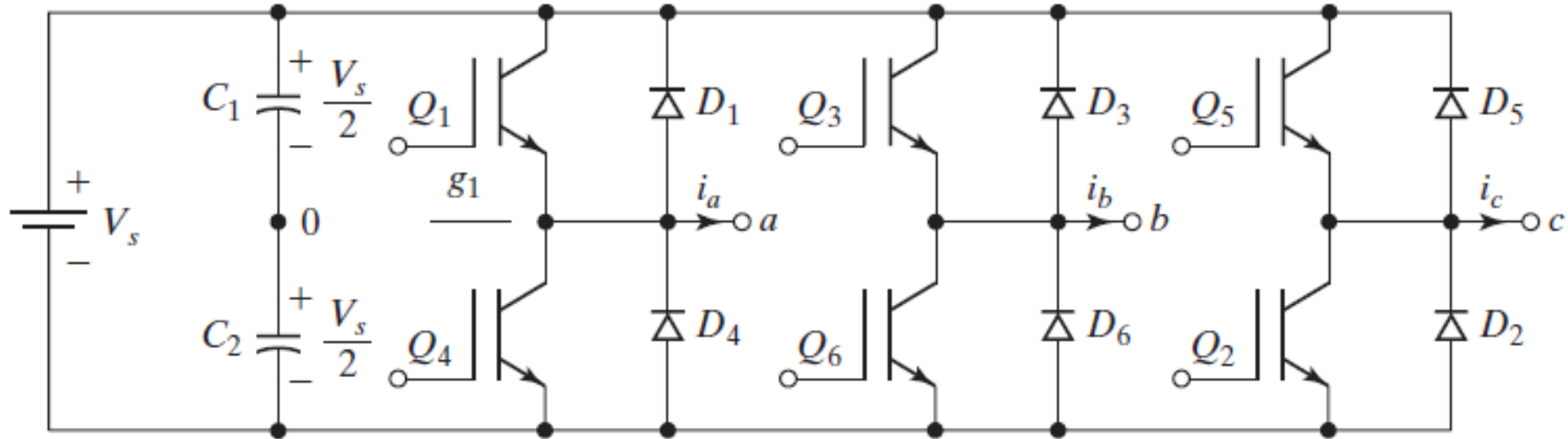


The peak value of the fundamental component of phase voltage is:

$$V_{1,peak,L-N} = \frac{2}{\pi} V_s$$



Voltage source inverter

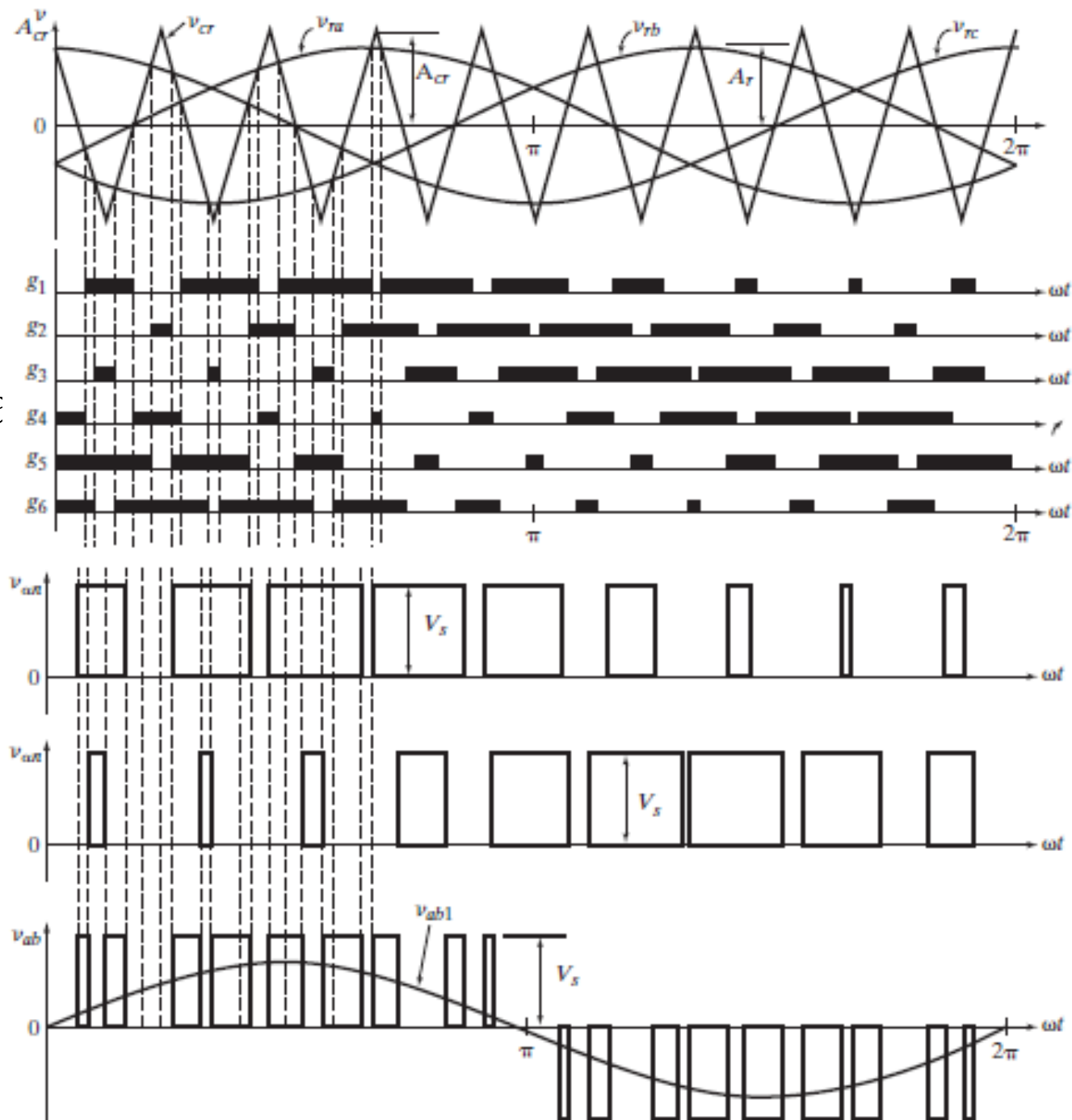


Sinusoidal Pulse Width Modulation (SPWM)

The peak value of the fundamental component of phase voltage is:

$$V_{1,peak,L-N} = \frac{MV_s}{2}$$

where M is the modulation index



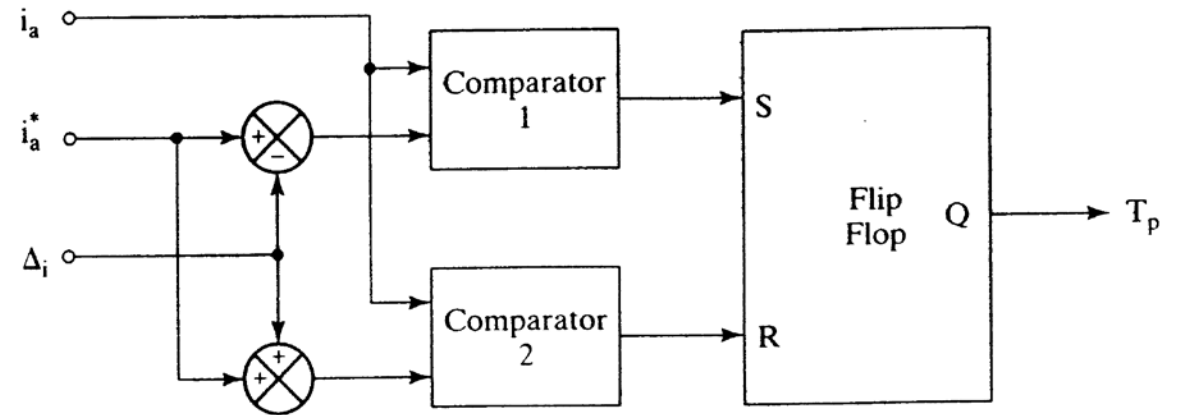
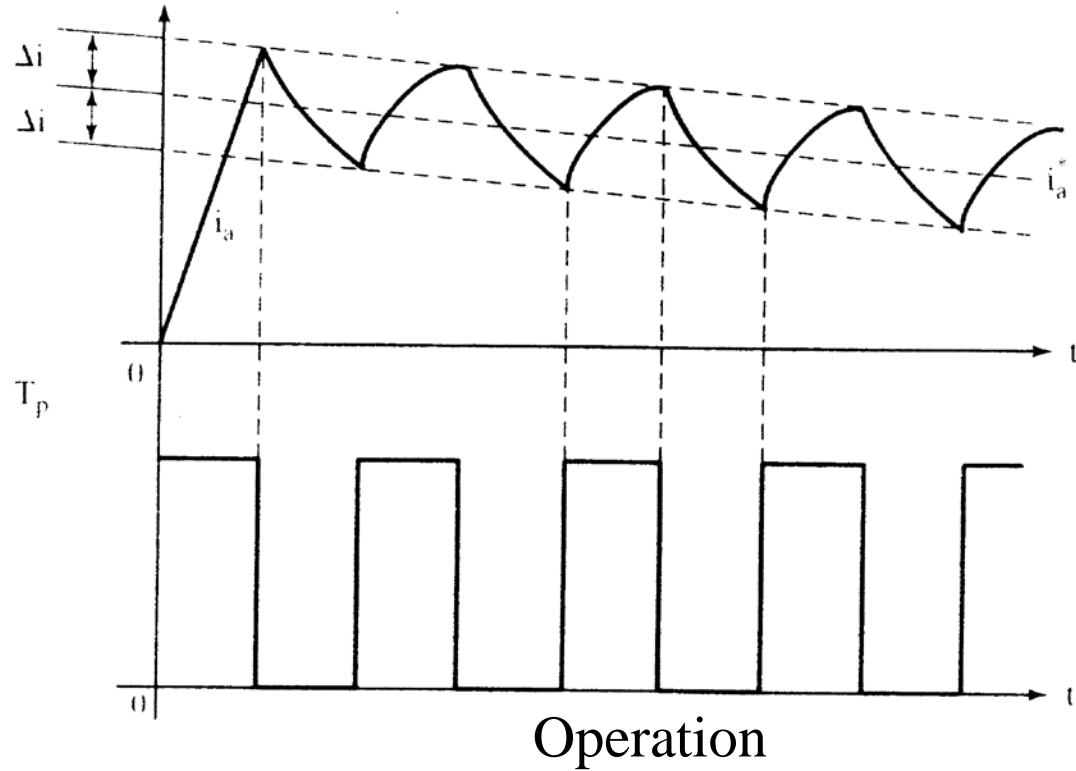
Hysteresis of Delta Modulation

It is an instantaneous current controller

The voltage applied to the load is determined by the following logic:

$$i_a \leq (i_a^* - \Delta i) \Rightarrow \text{Set } v_{an} = \frac{V_s}{2}$$

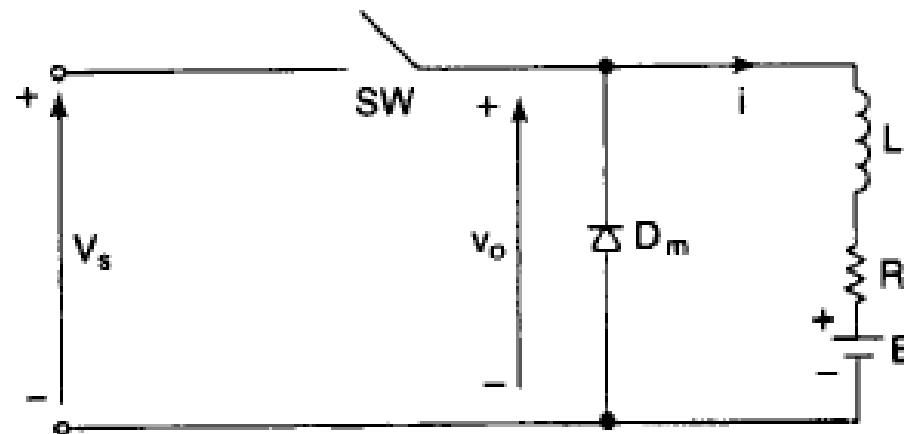
$$i_a \geq (i_a^* + \Delta i) \Rightarrow \text{Set } v_{an} = -\frac{V_s}{2}$$



Realization of hysteresis controller

1.3 DC-DC Converters (Chopper Circuits)

- They provide variable DC voltages for the drive of DC machines.
- The DC supply input to the inverter is derived either from a battery as in the case of electric vehicles or from a rectified AC source with uncontrolled rectifiers

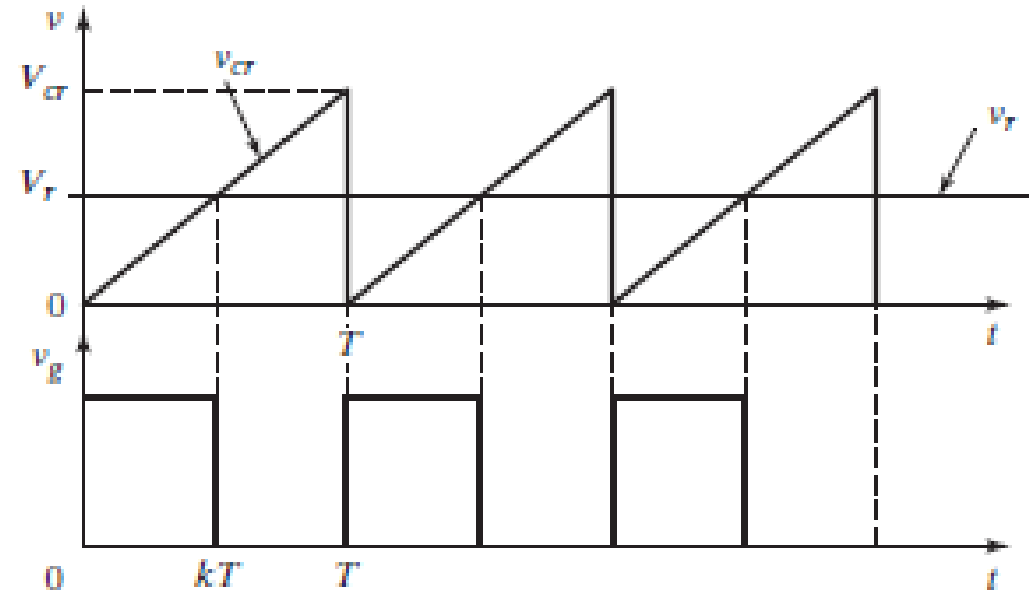


Pulse Width Modulation (PWM)

The peak value of the fundamental component of phase voltage is:

$$V_{o,avg} = kV_s$$

where k is the duty cycle



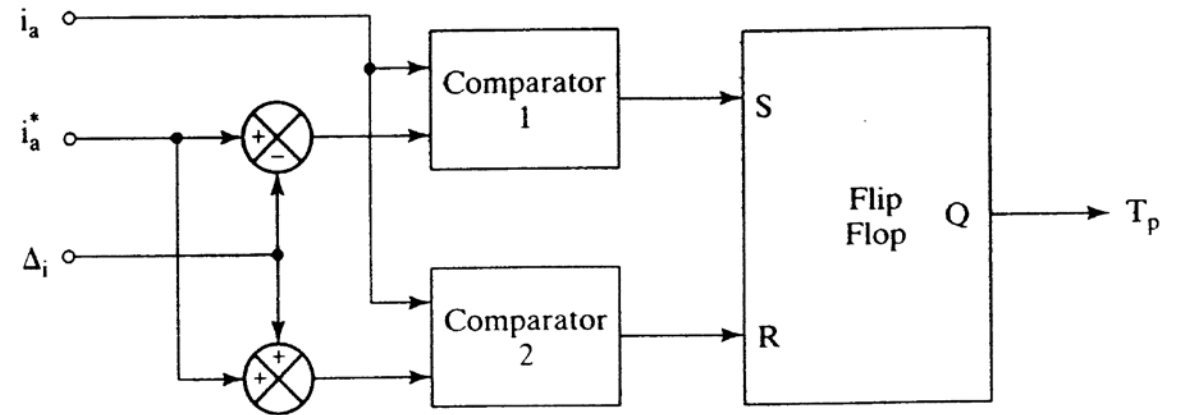
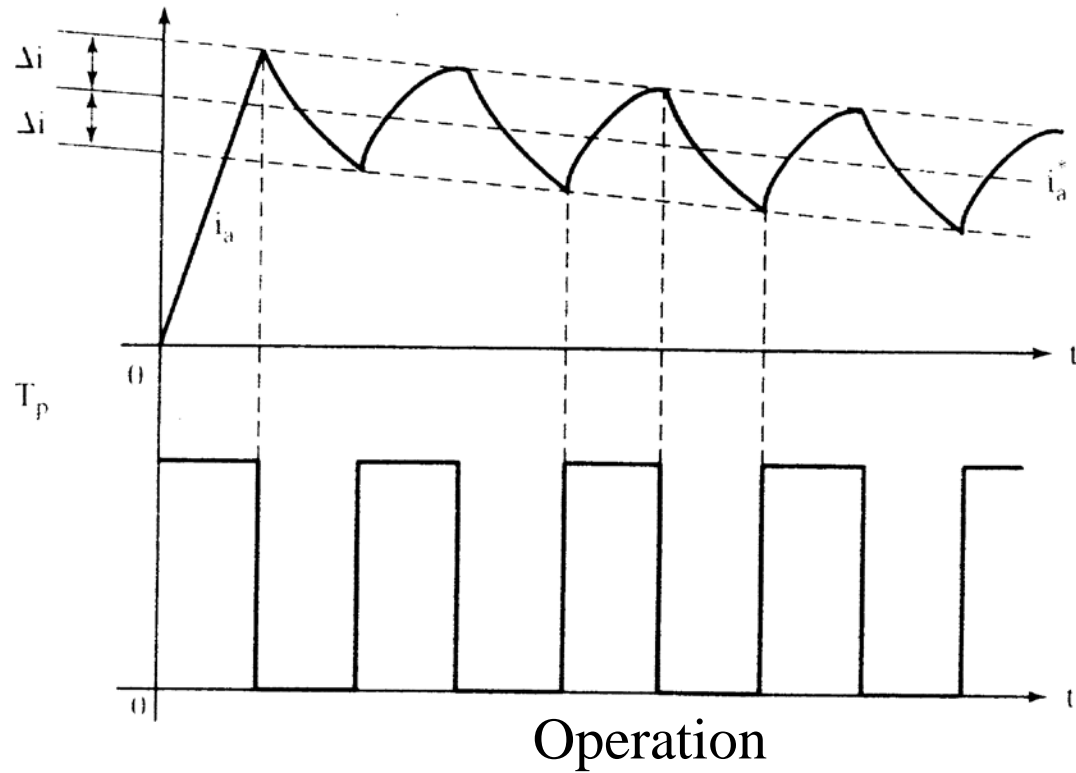
Hysteresis of Delta Modulation

It is an instantaneous current controller

The voltage applied to the load is determined by the following logic:

$$i_a \leq (i_a^* - \Delta i) \Rightarrow \text{Set } v_{an} = \frac{V_s}{2}$$

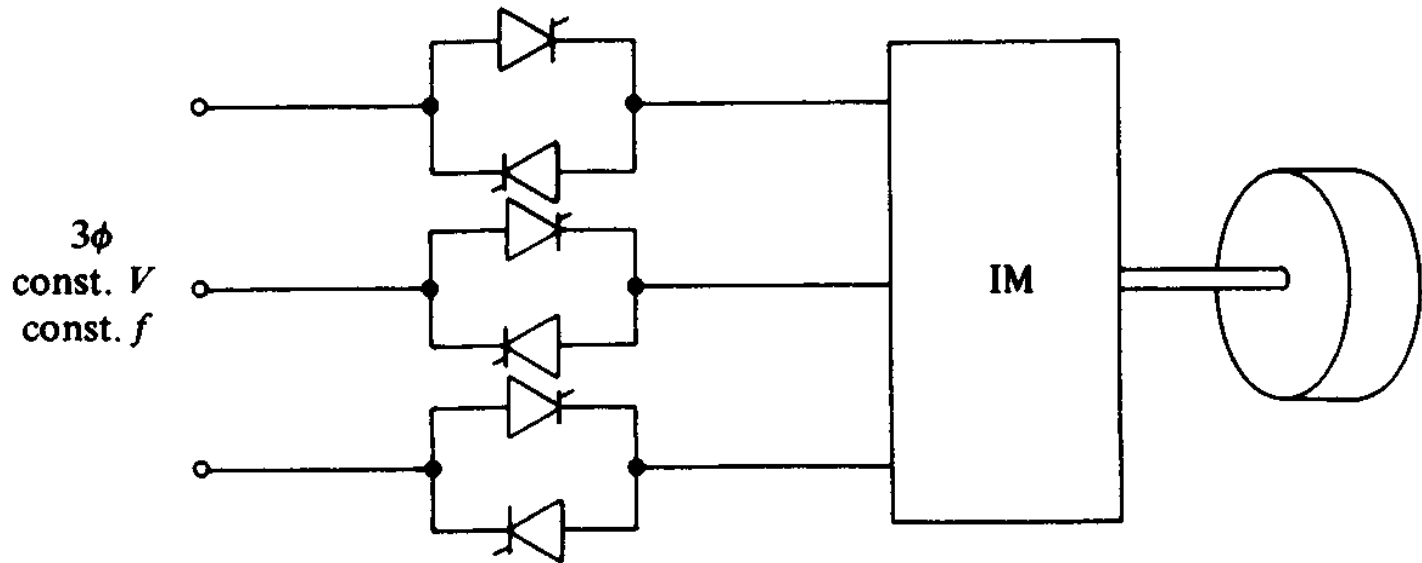
$$i_a \geq (i_a^* + \Delta i) \Rightarrow \text{Set } v_{an} = -\frac{V_s}{2}$$



Realization of hysteresis controller

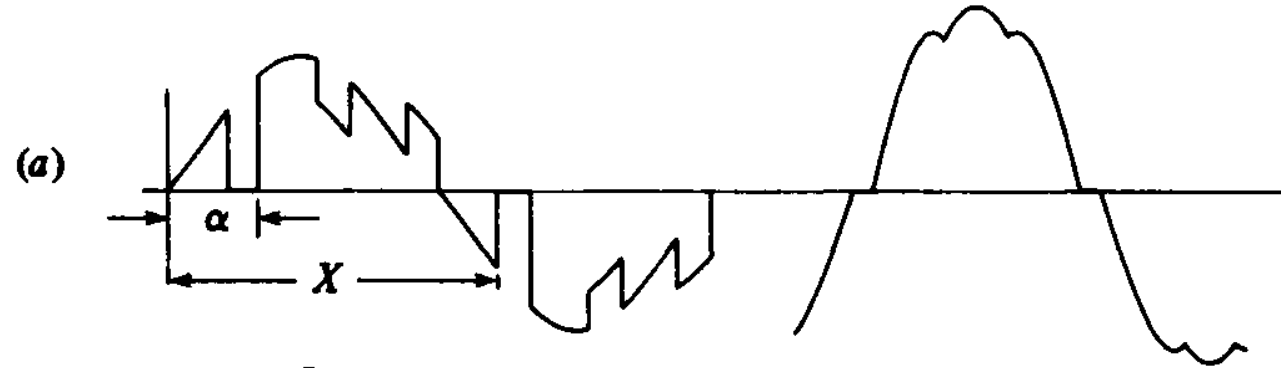
1.4 AC voltage controllers

- They provide variable AC voltages and currents at fixed frequency.
- The AC input terminals are connected to a three-phase AC source at fixed voltage and frequency
- Applications: Fan and Pump drives

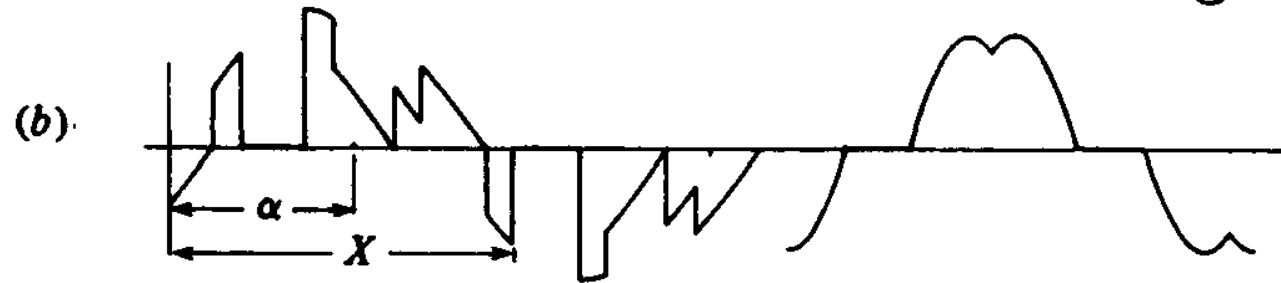


- The change of voltage is obtained at the expense of a low PF and considerable amount of harmonics
- The torque capabilities is also reduced
- The gate signals are synchronized to the phase voltages and shifted from each other by 60 degrees
- The power can flow only from a three-phase supply to the machine, and it can run only in one direction
- By using an appropriate firing angle, the motor can be soft started such that the motor starting current (starting torque) is restricted by motor voltage reduction, and hence increase the motor runtime

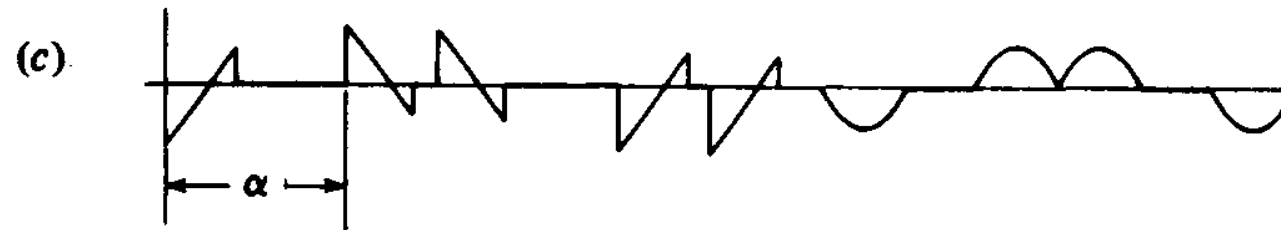
- The waveforms of motor voltages and currents are vary in three separate modes over the range of firing angle.
- If the firing angle is constant, the motor voltage and current waveforms vary with the motor power factor. Therefore, the analysis of this converter would be very complex because of interaction between the motor and controller
- The controller output voltage (input of motor) depends on both the state of controller and the state of the load. Therefore, simulation methods must be used.



(a) $\alpha = 60^\circ, \phi = 45^\circ$



(b) $\alpha = 120^\circ, \phi = 45^\circ$



(c) $\alpha = 120^\circ$, highly inductive.

(i) phase voltages

(ii) phase currents

2. Electrical AC Machines

- Types of AC machines: Induction and synchronous motors
- Types of DC machines: Separately, shunt, series, compound, permanent magnet (excited) DC motors
- A number of factors must be considered when you select an AC motor for a particular application
 - Cost
 - Power density, volume of motor
 - Thermal capacity
 - Torque-speed profile
 - Efficiency
 - Acceleration
 - Peak torque capability

- Motor rating

- For variable speed drive, the selection of motor torque and power ratings are not straightforward as in the case of constant speed drive
- In varying load case, the motor torque and power are selected on the basis of effective torque and power
- The effective torque is calculated from the load profile as:

$$T_{eff} = \sqrt{\frac{\sum_{i=1}^n T_i^2 t_i}{\sum_{i=1}^n t_i}}$$

where T_i is the average torque during the period t_i

3. Control system

- It controls the input of power converter to match the load and motor through the power converter
- The input of control system consists of:
 - Torque, flux, speed
 - The measured torque, flux, and speed for feedback control
 - Temperature feedback and instantaneous currents in the motor and/or converter
 - The gains of Proportional-Integral (PI) and/or Proportional-Resonant (PR) controllers

- The output of the control system is only the final gate signals which are directly issued to the gates of power devices
- Examples:
 - Microprocessors
 - Microcontrollers
 - Digital Signal Processors (DSPs)
 - Field Programmable Gate Arrays (FPGAs)

4. Load

- The torque developed by the motor is given by (Newton's 2nd Law)

$$T_{el} = T_l + J \frac{d\omega_m}{dt}$$

where

- T_{el} is the torque developed by the motor
- J is inertia seen by the motor's shaft
- ω_m is the motor's speed
- T_l is load torque

- The load torque consists of
 - Friction torque, T_F
 - Windage torque T_W
 - Torque required doing useful mechanical work, T_L
- The load torque is given by:

$$T_l = T_F + T_W + T_L$$

$$T_F = B\omega_m$$

$$T_W = C\omega_m^2$$

Load torque-speed characteristics

- $T_L = k$ or constant
 - Examples:
 - Machines work like shaping, cutting, grinding or shearing
 - Cranes during the hoisting
 - Conveyors
- Constant, $T_L = k \omega_m$
 - Examples:
 - Separately excited DC generator connected to a constant resistor

- $T_L = k \omega_m^2$
 - Examples:
 - Fans
 - Pumps
 - Compressors
 - Ship propellers
- Constant, $T_L = k/\omega_m$
 - Examples:
 - Lathes
 - Boring machines
 - Milling machines
 - Steel mill coiler

- $T_L = k_0 + k_1 \omega_m$
 - Examples:
 - Hoist
 - Elevator
- Constant, $T_L = k_0 + k_1 \omega_m + k_2 \omega_m^2$
 - Examples:
 - Compressor

Inertia

- The motor is usually connected to the load through a set of gears
- The gears have teeth ratio and can be treated as a torque transformer
- The gears are used to amplify the torque on the load side that is at lower speed compared to the motor's speed
- The gears can be modeled from the following facts:
 - The power handle by the gear is the same on both sides
 - Speed on each side is inversely proportional to the tooth number

$$\frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \tau$$

$$T_1\omega_1 = T_2\omega_2 \Rightarrow \frac{T_1}{T_2} = \frac{\omega_2}{\omega_1} = \frac{1}{\tau}$$

where

- N_1 and N_2 are the teeth numbers in the gear
- τ is gearbox ratio
- ω_1 is the motor's speed
- ω_2 is the load speed
- T_2 is the load torque
- T_1 is the load torque referred to the motor's shaft

- The load inertia reflected to the motor shaft can be calculated as:

$$\frac{1}{2} J_{L,ref} \omega_1^2 = \frac{1}{2} J_L \omega_2^2$$
$$J_{L,ref} = J_L \left(\frac{\omega_2}{\omega_1} \right)^2 = \frac{J_L}{\tau^2}$$

where

- J_L is the load inertia
- $J_{L,ref}$ is the load inertia reflected to the motor's shaft

- The total inertia seen by the motor is given by:

$$J = J_m + J_{L,ref} = J_m + \frac{J_L}{\tau^2}$$

where

- J_m is the motor's inertia for the shaft

- Maximum power transfer occurs if the load inertia reflected to the motor shaft is made to match the motor inertia. In this case, maximum acceleration of the load will result

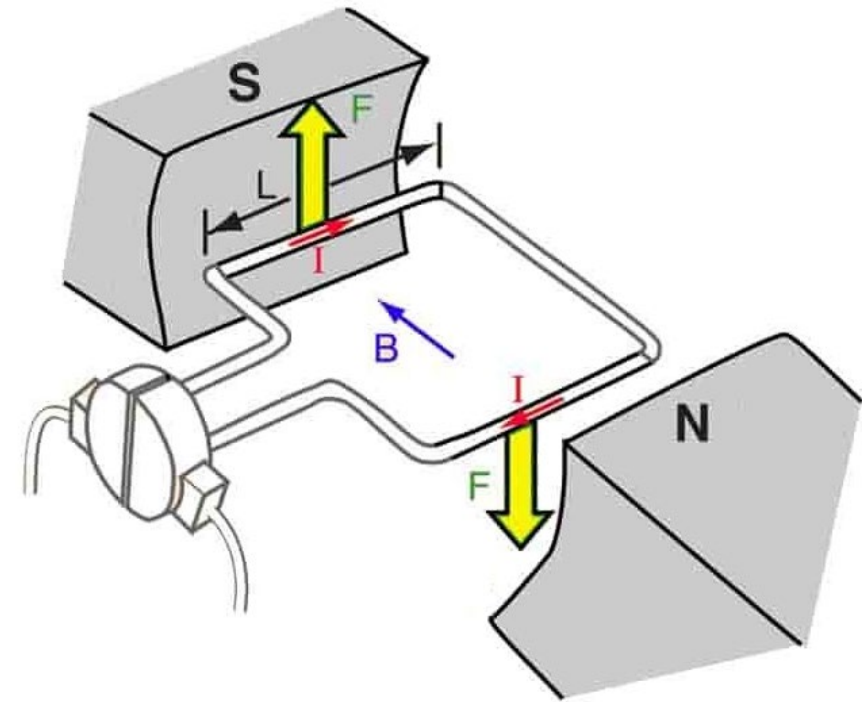
$$T_{el} - T_l = J \frac{d\omega_1}{dt} \Rightarrow T_{el} - T_l = \left(J_m + \frac{J_L}{\tau^2} \right) \tau \frac{d\omega_2}{dt}$$

$$T_{el} - T_l = \left(J_m \tau + \frac{J_L}{\tau} \right) \alpha_2 \Rightarrow \frac{T_{el} - T_l}{\alpha_2} = F(\tau) = J_m \tau + \frac{J_L}{\tau}$$

$$\frac{\partial F(\tau)}{\partial \tau} = 0 \Rightarrow J_m = \frac{J_L}{\tau^2} = J_{L,ref}$$

DC Machine Drive

- Theory of operation of DC motor
 - The electric current, i_a , passes through the armature winding via a commutator and brushes
 - When i_a passes through the armature winding in a magnetic field, a magnetic force, F_e , is induced
 - The magnetic force induces a torque, which turns the DC motor



- Equivalent circuit of DC motor armature

- R_a and L_a are the resistance and self-inductance of armature windings
- e_a is the back emf voltage or induced voltage

$$e_a = K\phi_f\omega_m; \quad \phi_f = Ci_f$$

where

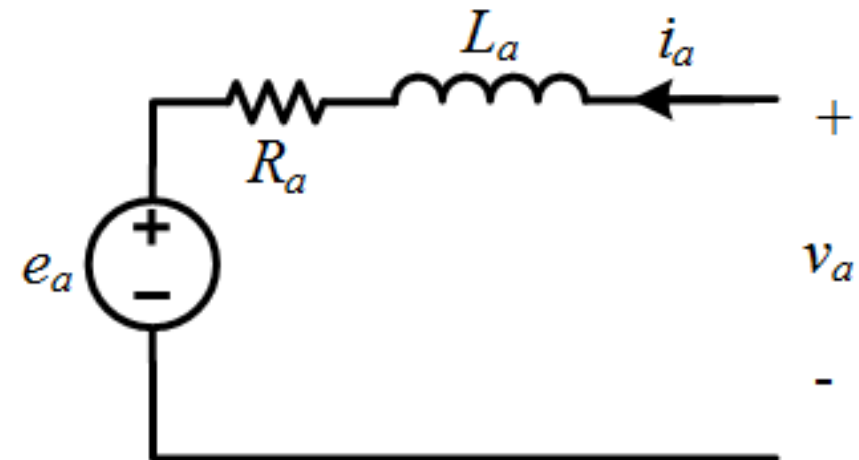
K is constant depends on machine's structure

ϕ_f is the machine's flux

ω_m is the machine's speed

C is constant (slop of magnetizing curve)

i_f is the field current



- Torque equation

KVL in the armature circuit

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a$$

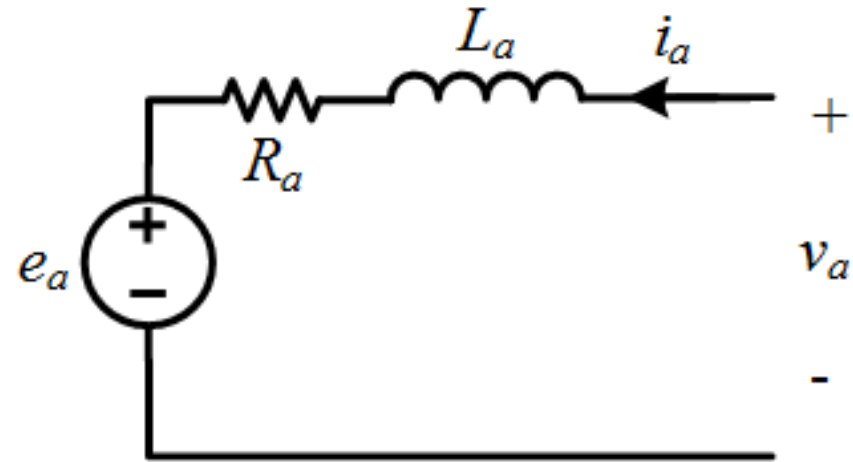
Under steady state operation,

$$v_a = R_a i_a + e_a$$

$$\underbrace{v_a i_a}_{\text{Total input Power}} = \underbrace{R_a i_a^2}_{\text{Armature copper losses}} + \underbrace{e_a i_a}_{\text{Air-gap power}}$$

The air-gap power, P_a , is the effective power that has been transferred to mechanical power

$$P_a = e_a i_a = T_{el} \omega_m \Rightarrow T_{el} = \frac{e_a i_a}{\omega_m} = \frac{K \phi_f \omega_m i_a}{\omega_m} \Rightarrow T_{el} = K \phi_f i_a$$



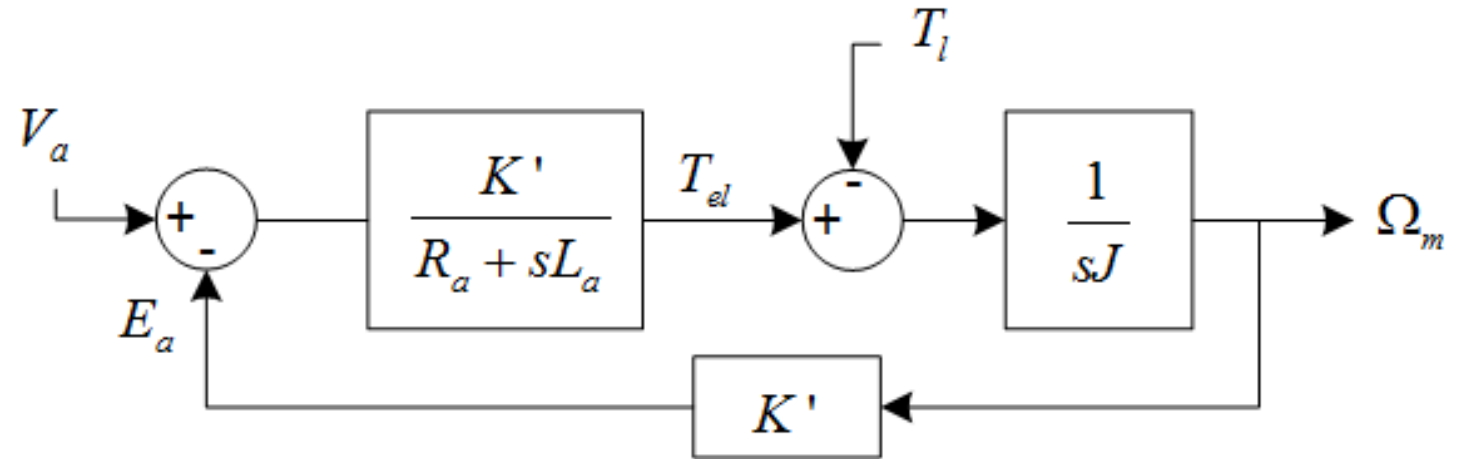
- Block diagram and transfer function of DC machine
 - Machine equations in time-domain

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a; e_a = K \phi_f \omega_m = K' \omega_m; T_{el} = K \phi_f i_a = K' i_a; T_{el} = T_l + J \frac{d\omega_m}{dt}$$

- Machine equations in s -domain

$$V_a = (R_a + sL_a) I_a + E_a; E_a = K' \Omega_m; T_{el} = K' I_a; T_{el} = T_l + Js \Omega_m$$

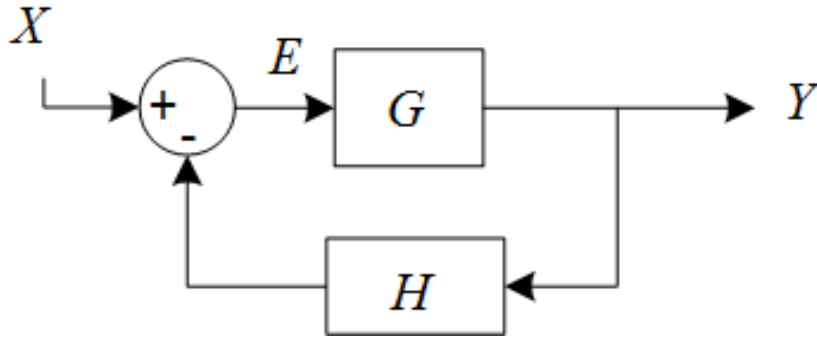
- Block diagram



$$\Omega_m = T_1 V_a + T_2 T_l$$

- T_1 is the transfer function when $T_l = 0$
- T_2 is the transfer function when $V_a = 0$

- Closed loop system



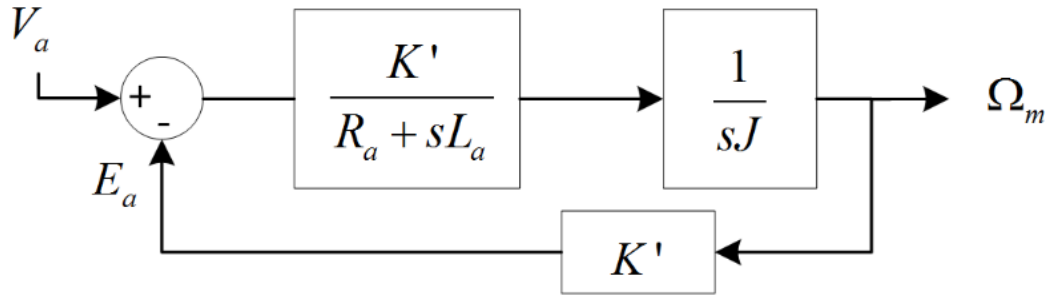
$$Y = GE$$

$$Y = G(X - HY) = GX - GHY$$

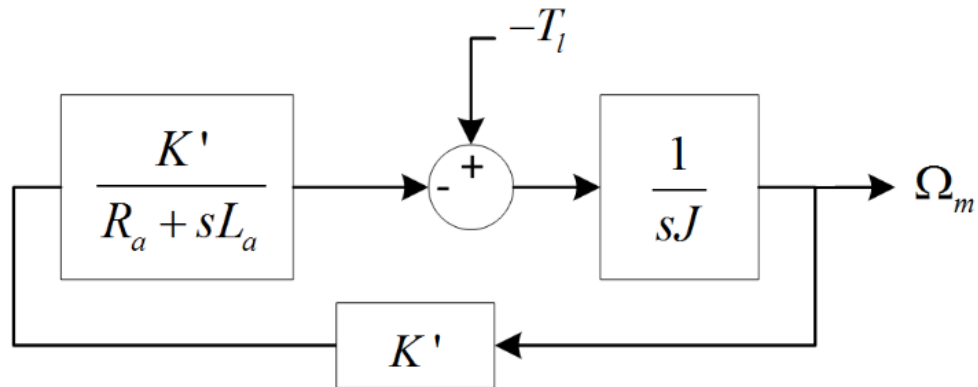
$$Y = TX; \quad T = \frac{G}{1 + GH}$$

- G is the direct transfer function
- H is the feedback transfer function

- $T_1: T_l = 0$



- $T_2: V_a = 0$



$$T_1 = \frac{G_1}{1 + G_1 H_1};$$

$$G_1 = \frac{K'}{sJ(R_a + sL_a)}; \quad H_1 = K'$$

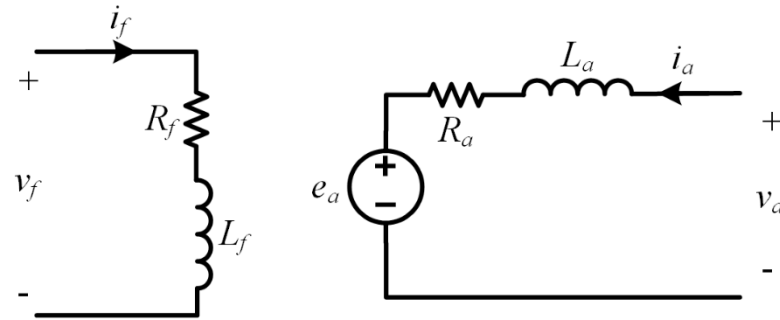
$$T_2 = \frac{-G_2}{1 + G_2 H_2};$$

$$G_2 = \frac{1}{sJ}; \quad H_2 = \frac{K'^2}{R_a + sL_a}$$

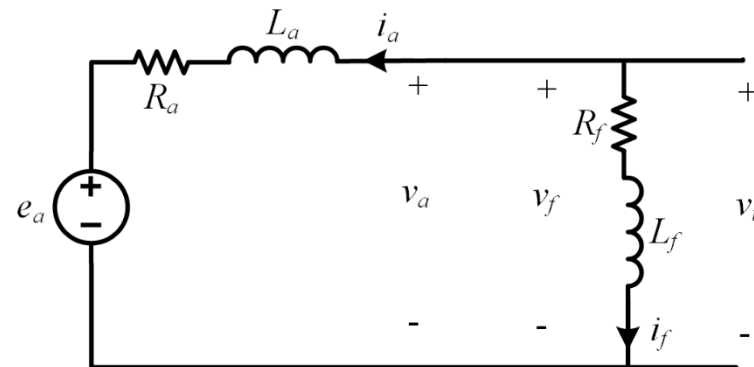
- Field Excitation
 - Separately excited DC machine
 - Shunt excited DC machine
 - Series excited DC machine
 - Compound DC machine
 - Permanent magnet DC machine (PMDC)

Steady state torque-speed relationship

- Separately and shunt excited DC machines



Separately

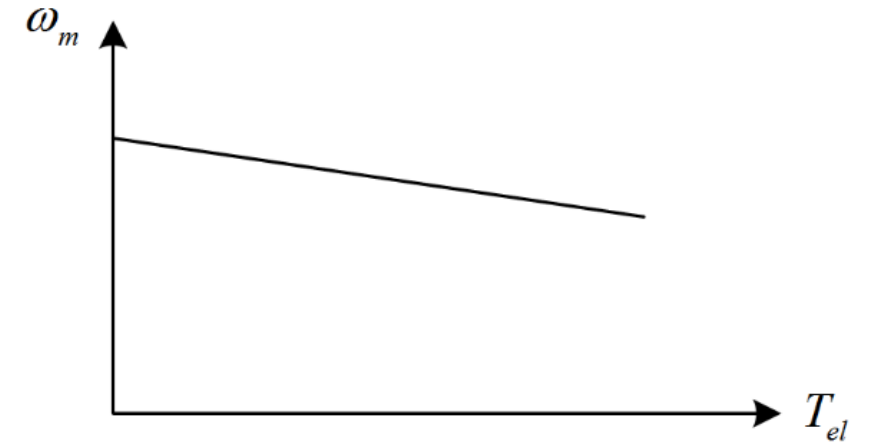


Shunt

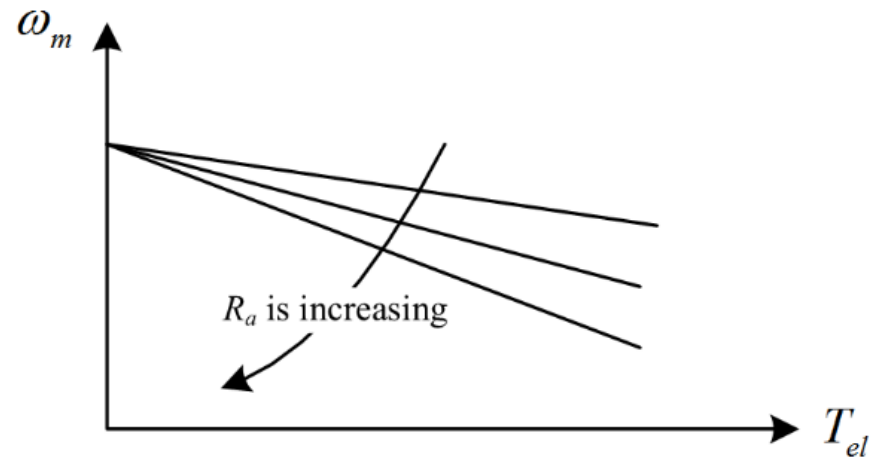
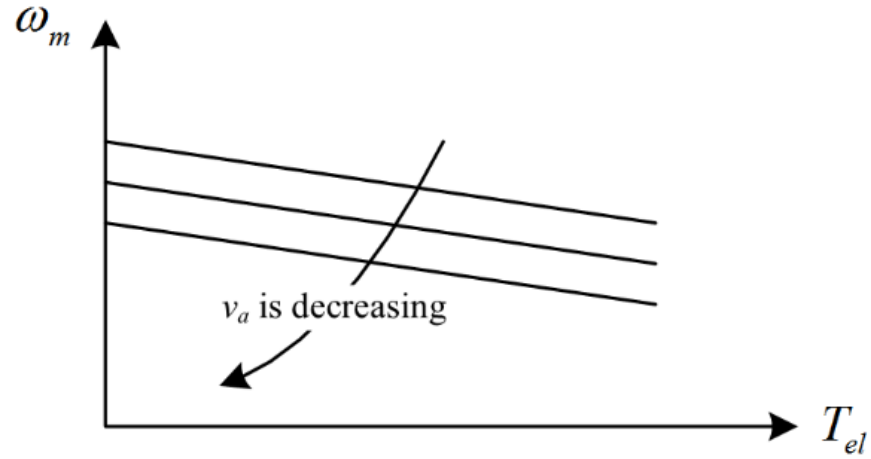
$$v_a = R_a i_a + e_a; \quad e_a = K\phi_f \omega_m \Rightarrow v_a = R_a i_a + K\phi_f \omega_m$$

$$T_{el} = K\phi_f i_a \Rightarrow i_a = \frac{T_{el}}{K\phi_f}$$

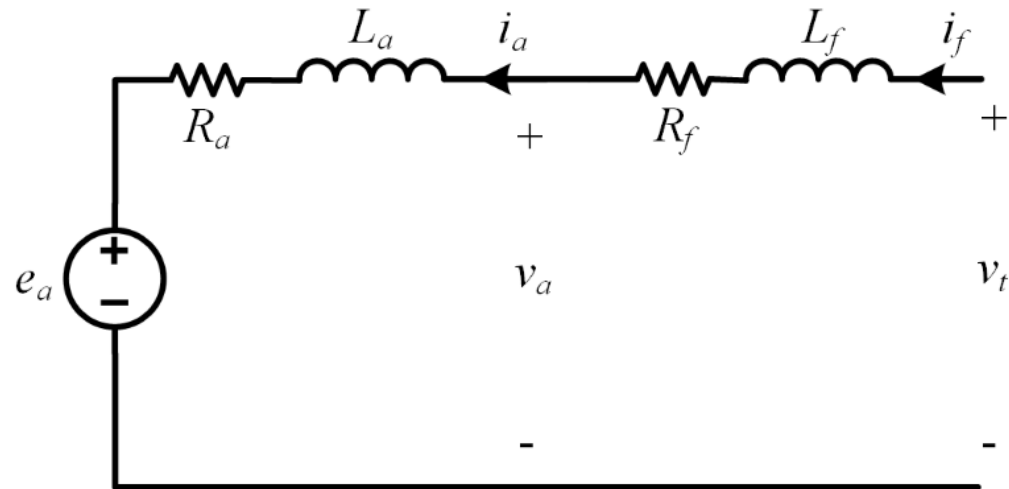
$$v_a = R_a \frac{T_{el}}{K\phi_f} + K\phi_f \omega_m \Rightarrow \omega_m = \frac{v_a}{K\phi_f} - \frac{R_a}{(K\phi_f)^2} T_{el}$$



- Methods of speed control
 - v_a control
 - R_a control



- Series excited DC machines

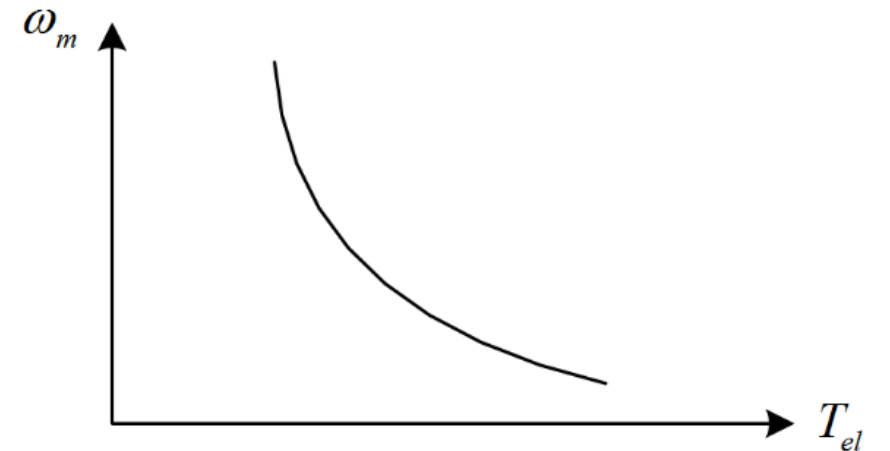


$$v_t = (R_a + R_f)i_a + e_a; \quad e_a = K\phi_f\omega_m; \quad \phi_f = Ci_f; \quad i_f = i_a$$

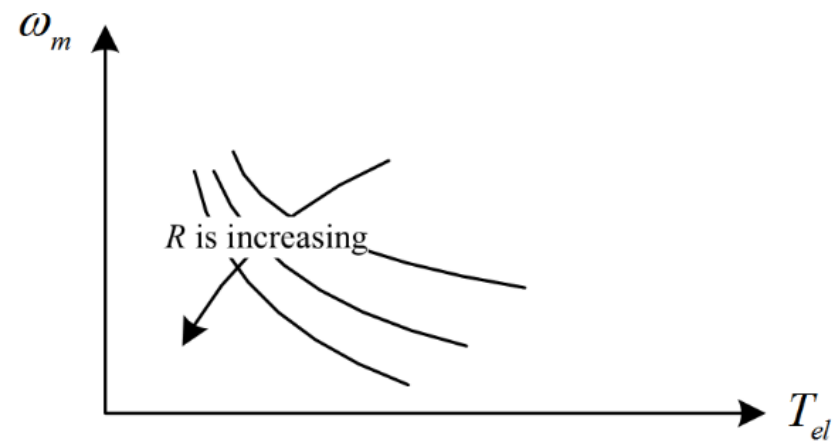
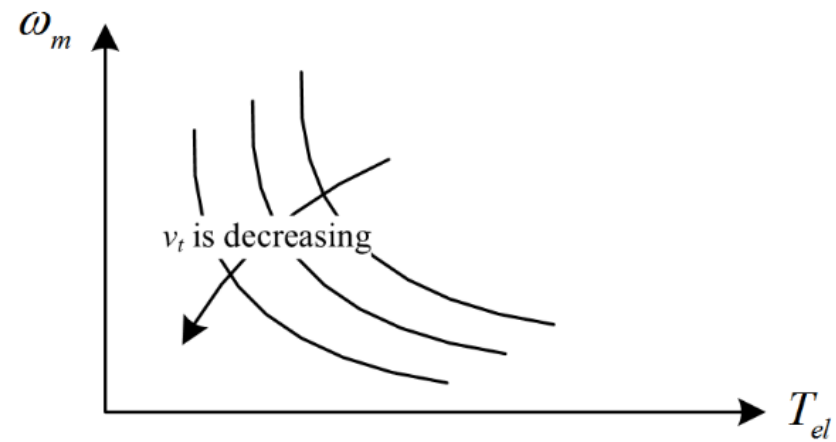
$$\Rightarrow v_t = Ri_a + KCi_a\omega_m$$

$$T_{el} = K\phi_f i_a \Rightarrow T_{el} = KCi_a^2 \Rightarrow i_a = \sqrt{\frac{T_{el}}{KC}}$$

$$v_t = \frac{R}{\sqrt{KC}}\sqrt{T_{el}} + \sqrt{KC}\sqrt{T_{el}}\omega_m \Rightarrow \omega_m = \frac{v_t}{\sqrt{KC}}\frac{1}{\sqrt{T_{el}}} - \frac{R}{KC}$$



- Methods of speed control
 - v_t control
 - R control

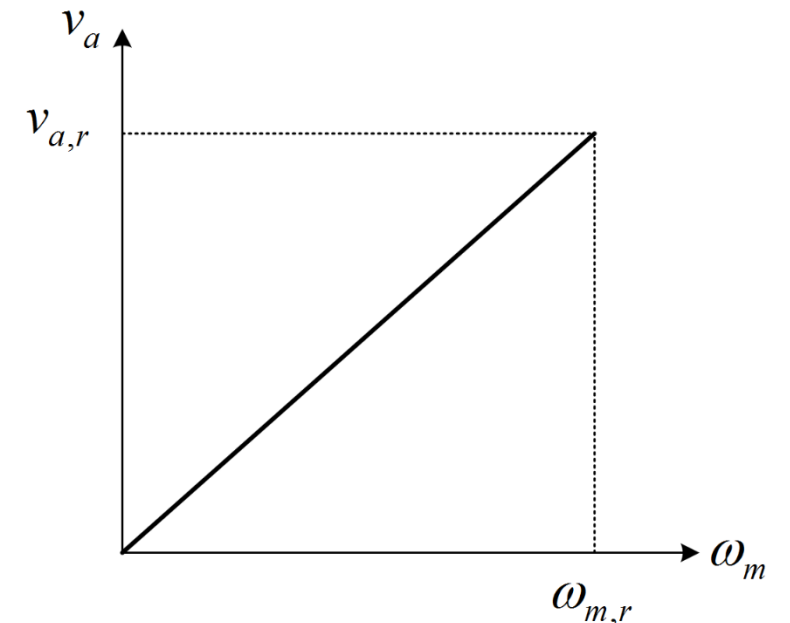


- Principle of DC machine drive

$$v_a = R_a i_a + K \phi_f \omega_m \Rightarrow \omega_m = \frac{v_a - R_a i_a}{K \phi_f}$$

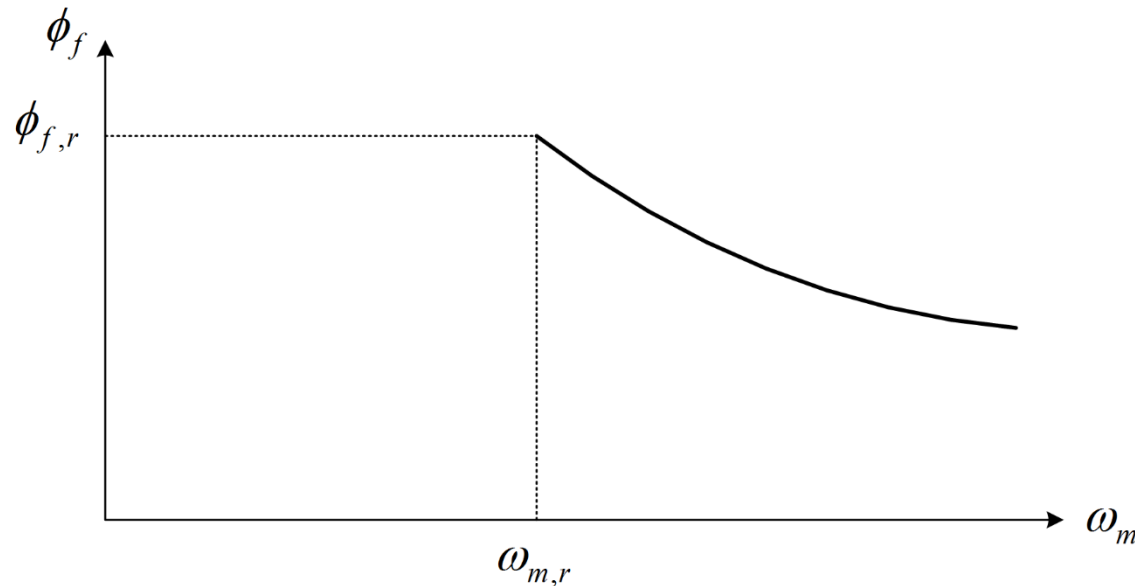
- Armature control: Ideal for speeds lower than motor's rated speed, $\omega_{m,r}$.

$$\phi_f = \phi_{f,r} \Rightarrow \omega_m = \frac{v_a - R_a i_a}{K \phi_{f,r}} \approx \frac{v_a}{K \phi_{f,r}} \Rightarrow \omega_m \propto v_a$$



- Field control: Ideal for speeds above than motor's rated speed, $\omega_{m,r}$.

$$v_a = v_{a,r} \Rightarrow \omega_m \approx \frac{v_{a,r}}{K\phi_f} \Rightarrow \phi_f \propto \frac{1}{\omega_m} \Rightarrow \phi_f = \phi_{f,r} \frac{\omega_{m,r}}{\omega_m}$$



- Armature and Field control: Assume that $i_a = i_{a,r}$

When $\omega_m \leq \omega_{m,r}$

$$\phi_f = \phi_{f,r}$$

$$T_{el} = K \phi_{f,r} i_{a,r} = \text{Constant}$$

$$P_a = K \phi_{f,r} i_{a,r} \omega_m$$

Constant torque region

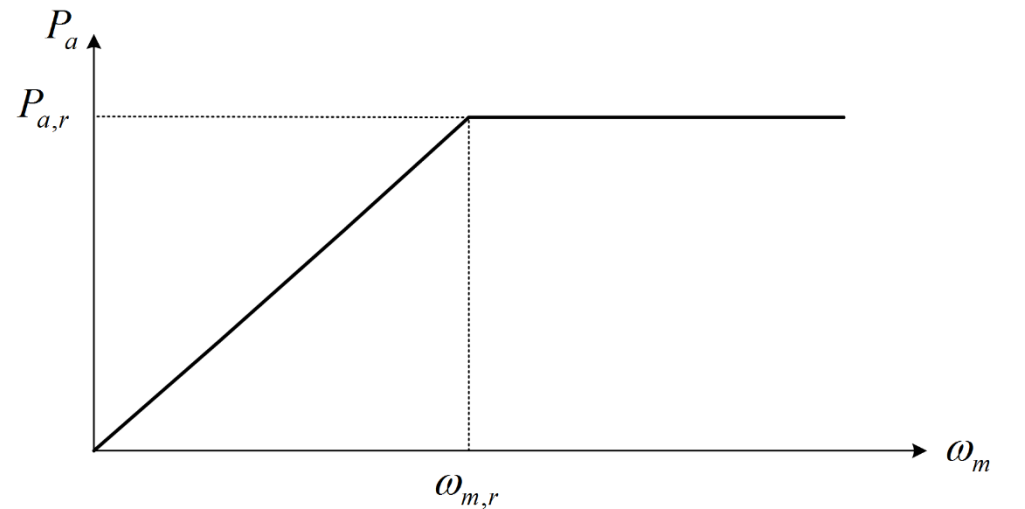
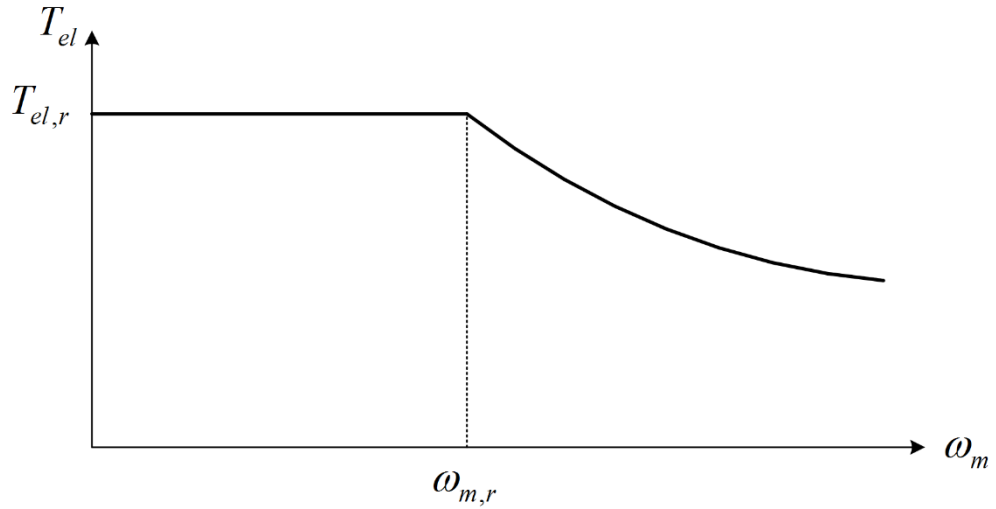
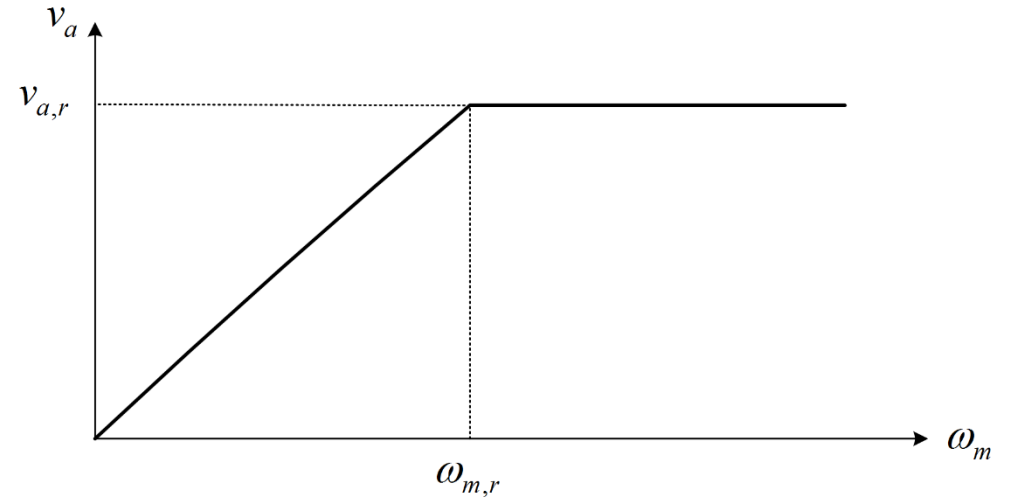
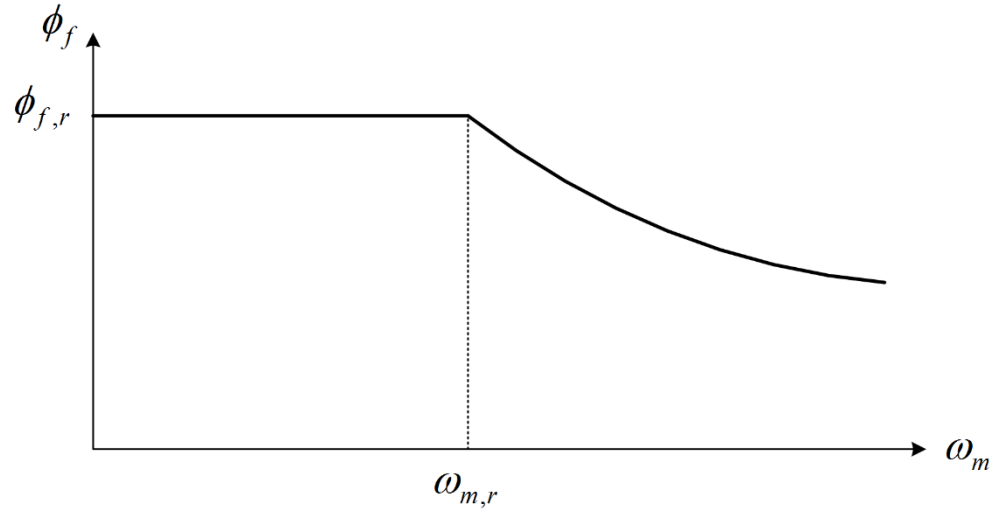
When $\omega_m \geq \omega_{m,r}$

$$\phi_f = \phi_{f,r} \frac{\omega_{m,r}}{\omega_m}$$

$$T_{el} = K \phi_{f,r} \frac{\omega_{m,r}}{\omega_m} i_{a,r}$$

$$P_a = K \phi_{f,r} i_{a,r} \omega_{m,r} = \text{Constant}$$

Constant power region



- Braking and 4-quadrant operation

- Braking

The machine is made to work as a generator producing a torque opposite to the motoring torque

$$T_{el} = T_l + J \frac{d\omega_m}{dt}$$

$$\omega_m > 0 \ \& \ (T_{el} - T_l) > 0 \Rightarrow \text{Forward regeneration}$$

$$T_{el} = T_l + J \frac{d\omega_m}{dt} \Rightarrow \Delta t = J \frac{\Delta\omega_m}{T_{el} - T_l}$$

Δt : stopping time

Why do we need braking?

1. Reducing Δt
2. Achieving quick and smooth stops
3. Achieving accurate stops
4. Holding the speed within safe limit

Braking methods:

1. *Regenerative braking*: generated electrical power is usefully employed
2. *Dynamic braking*: it is an inefficient way of braking
3. *Plugging braking*: it is a highly inefficient way of braking

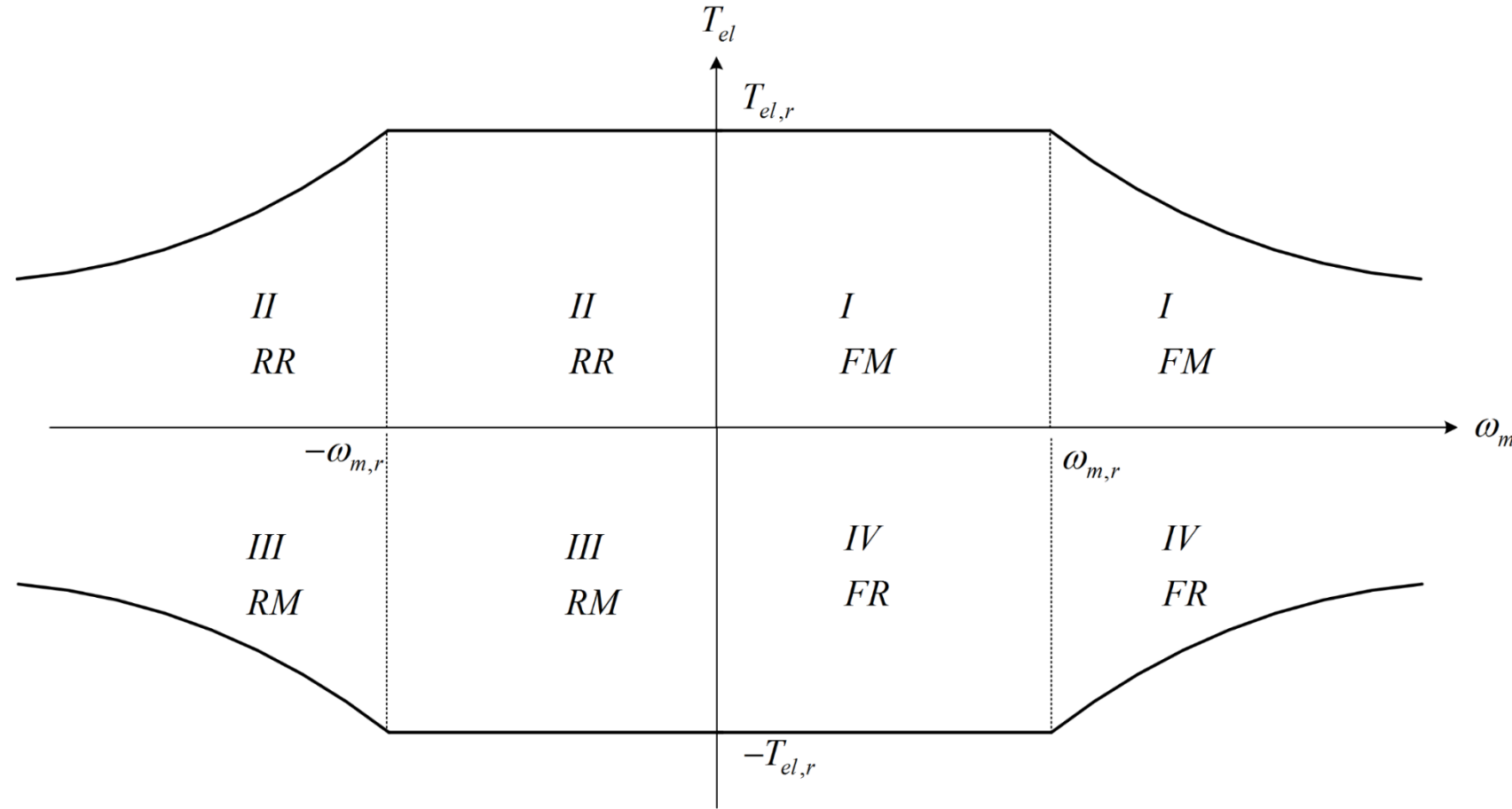
- 4-quadrant operation

FM: Forward Motoring

FR: Forward Regeneration

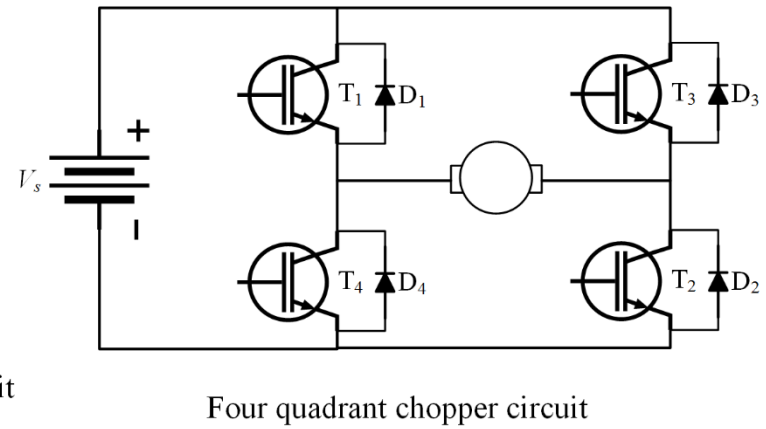
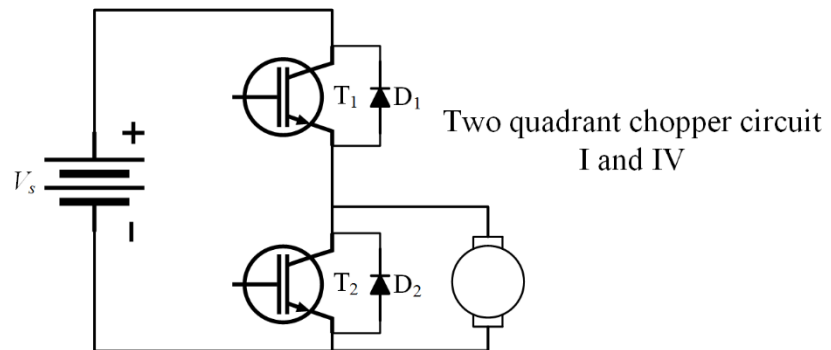
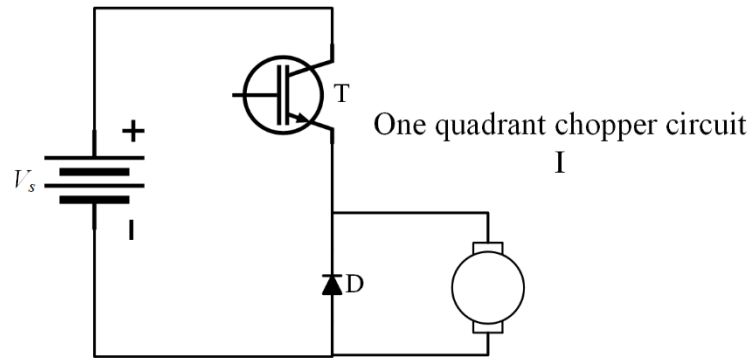
RM: Reverse Motoring

RR: Reverse Regeneration

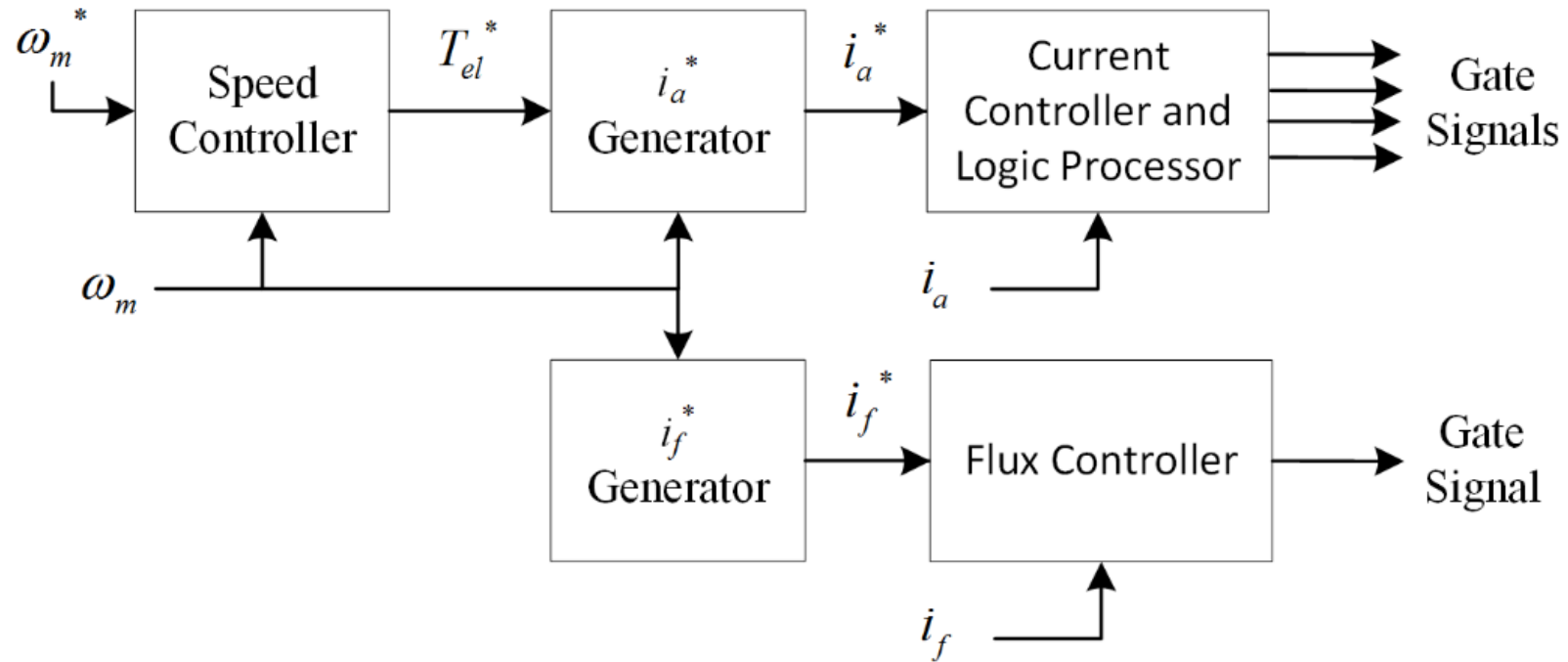


Function	Quadrant	Speed	Torque	Voltage	Current	Power
FM	I	+	+	+	+	+
FR	IV	+	-	+	-	-
RM	III	-	-	-	-	+
RR	II	-	+	-	+	-

• DC machine drive using chopper circuits



Block diagram of control system



- i_f^* Generator

$$\omega_m \leq \omega_{m,r} \Rightarrow i_f = i_{f,r}$$

$$\omega_m > \omega_{m,r} \Rightarrow i_f = i_{f,r} \frac{\omega_{m,r}}{\omega_m}$$

- i_a^* Generator

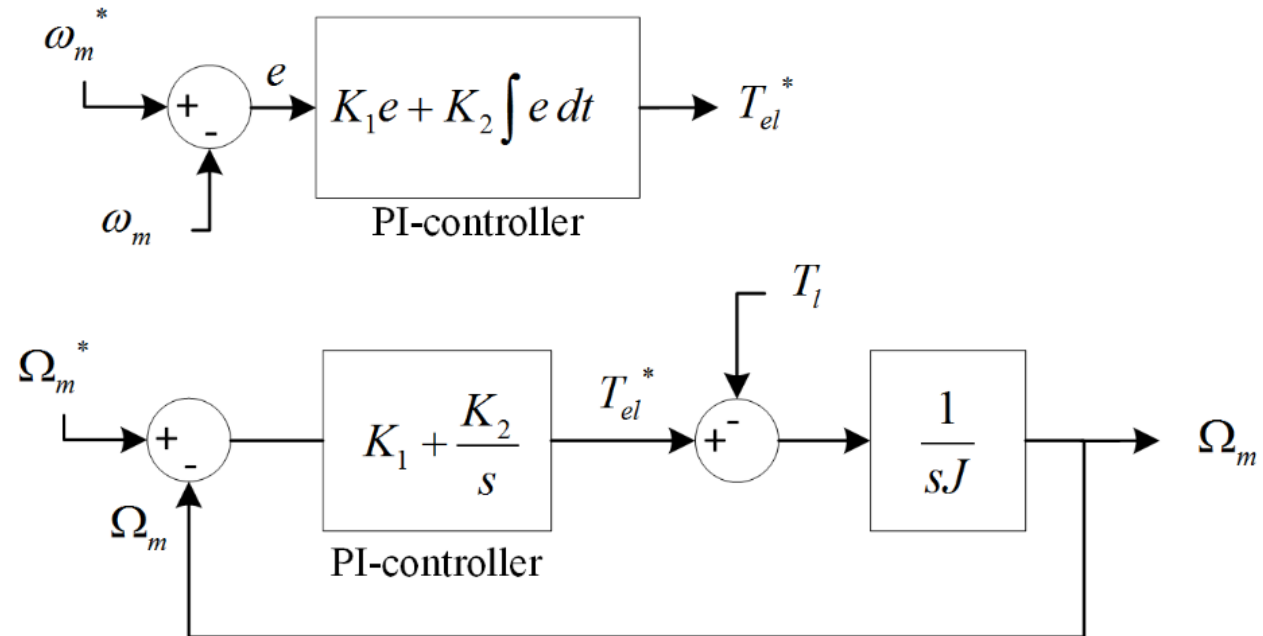
$$T_{el} = K \phi_f i_a \Rightarrow i_a^* = \frac{T_{el}^*}{K \phi_f}$$

$$\omega_m \leq \omega_{m,r} \Rightarrow \phi_f = \phi_{f,r} \Rightarrow i_a^* = \frac{1}{K \phi_{f,r}} T_{el}^*$$

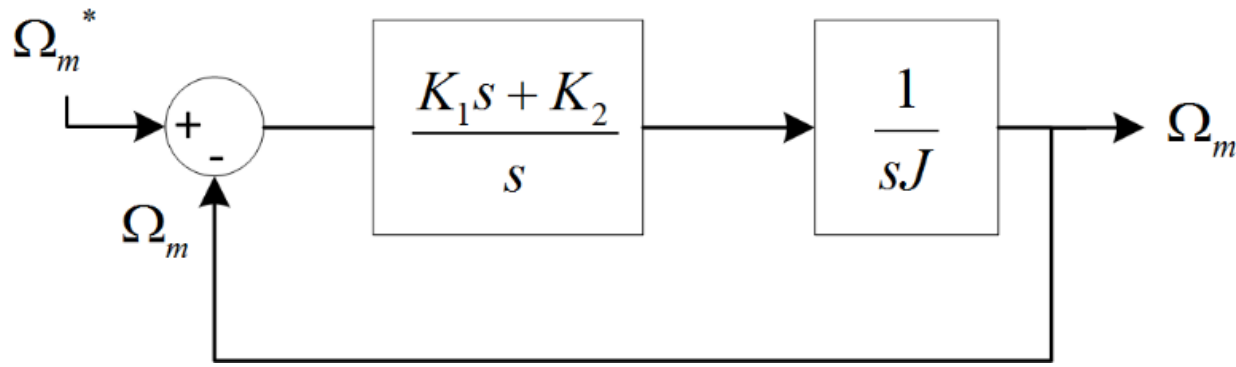
$$\omega_m > \omega_{m,r} \Rightarrow \phi_f = \phi_{f,r} \frac{\omega_{m,r}}{\omega_m} \Rightarrow i_a^* = \frac{\omega_m}{K \phi_{f,r} \omega_{m,r}} T_{el}^*$$

- Speed Controller: It is designed using Newton's 2nd law

$$T_{el} = T_l + J \frac{d\omega_m}{dt} \xrightarrow{\text{Laplace}} T_{el} = T_l + Js\Omega_m$$



- $T_1: T_l = 0$



$$T_1 = \frac{G_1}{1 + G_1 H_1};$$

$$G_1 = \frac{K_1 s + K_2}{J s^2}; \quad H_1 = 1$$

$$T_1 = \frac{K_1}{J} \frac{s + \frac{K_2}{K_1}}{\underbrace{s^2 + \frac{K_1}{J}s + \frac{K_2}{J}}_{s^2 + 2\xi\omega_n s + \omega_n^2}};$$

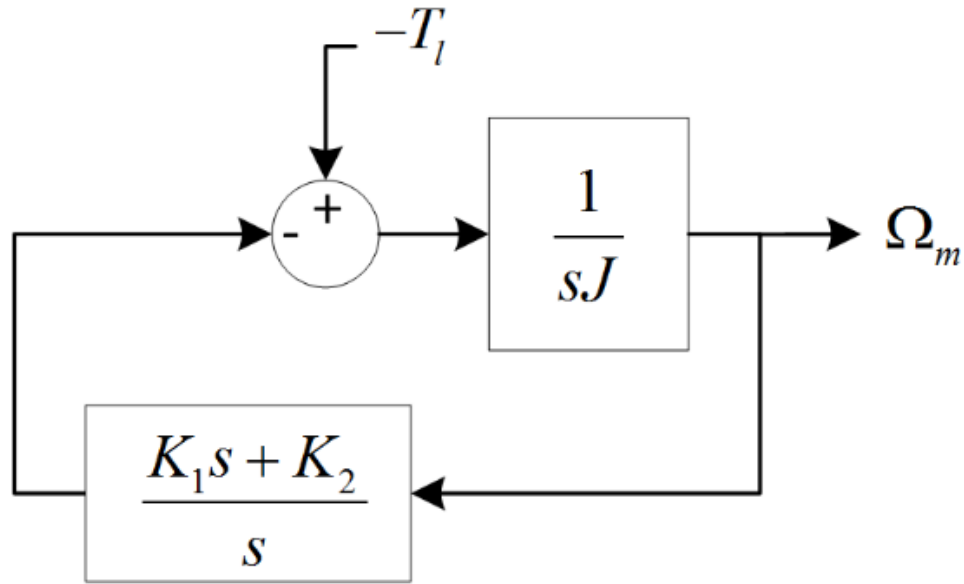
ξ : Damping ratio, ω_n : Natural frequency

$\xi < 1 \rightarrow$ Under-damped

$\xi = 1 \rightarrow$ Critically damped

$\xi > 1 \rightarrow$ Over-damped

- $T_2: \Omega_m^* = 0$



$$T_2 = \frac{-G_2}{1 + G_2H_2};$$

$$G_2 = \frac{1}{Js}; \quad H_2 = \frac{K_1s + K_2}{s}$$

$$T_2 = -\frac{1}{J} \frac{s}{s^2 + \frac{K_1}{J}s + \frac{K_2}{J}};$$

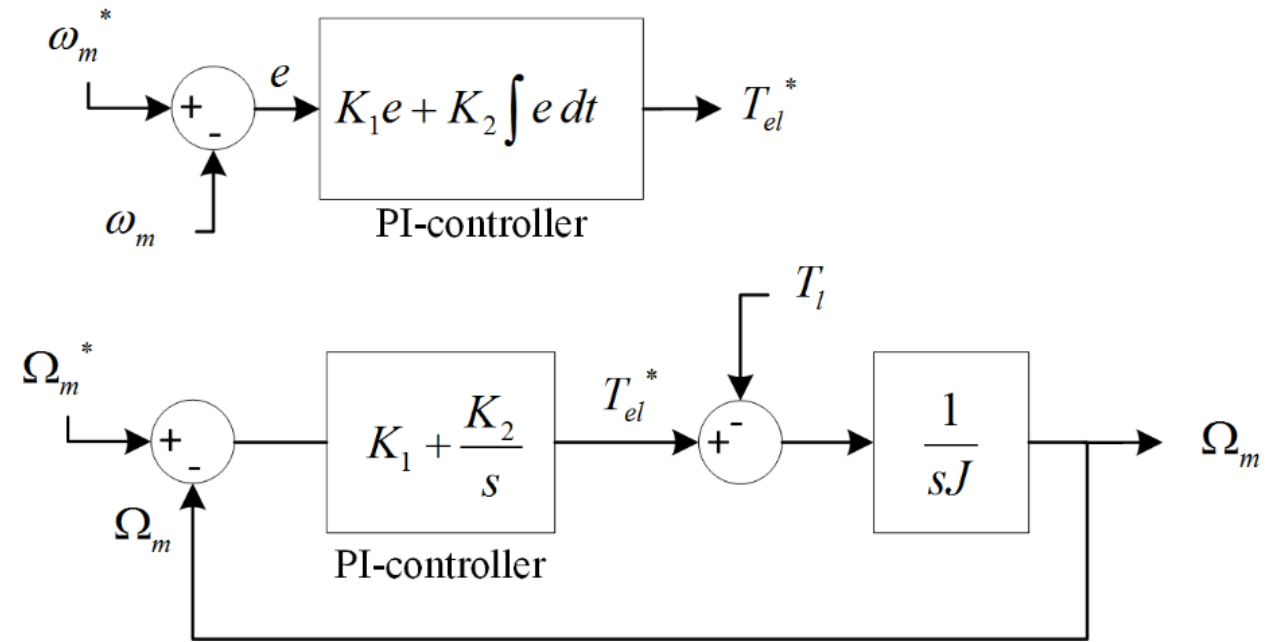
$$\Omega_m = T_1 \Omega_m^* + T_2 T_l$$

$$\omega_{m,ss} = \lim_{s=0} s \Omega_m = \lim_{s=0} s T_1 \Omega_m^* + \lim_{s=0} s T_2 T_l$$

Assume $\omega_m^* = Au(t)$, $T_l = Bu(t)$

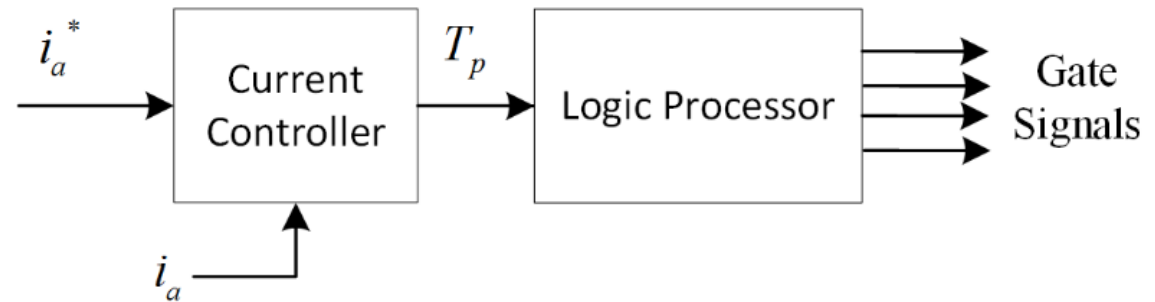
$$\Omega_m^* = \frac{A}{s}, T_l = \frac{B}{s}$$

$$\omega_{m,ss} = A \lim_{s=0} T_1 + B \lim_{s=0} T_2 = A$$



- Current Controller and Logic Processor:

- PWM current controller
- Hysteresis current controller



T_p is called the on-time signal

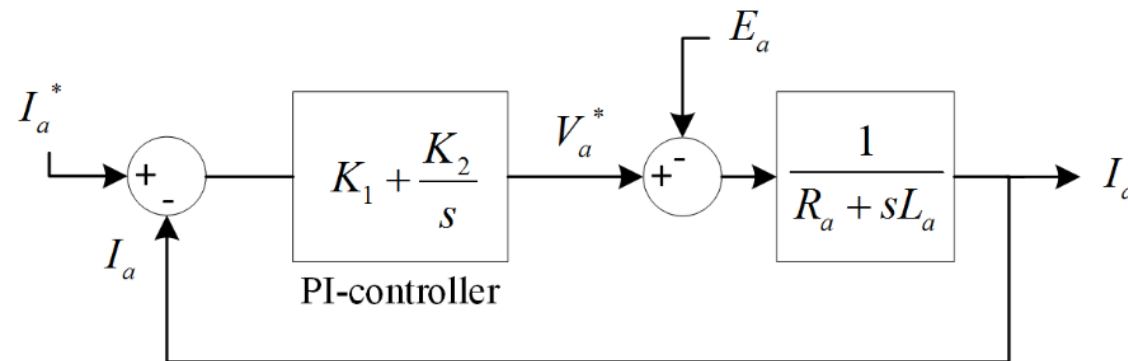
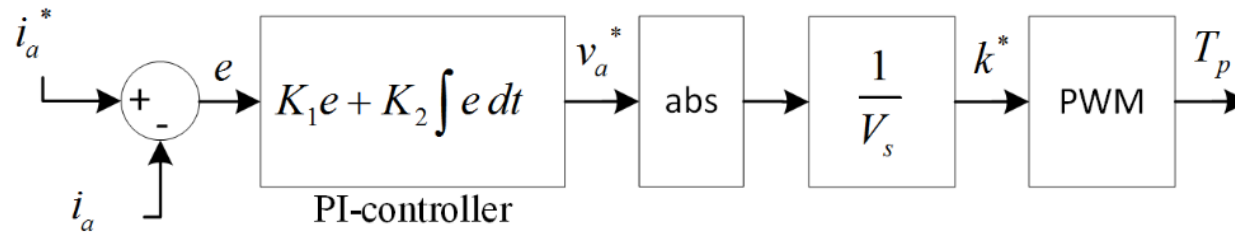
The logic processor determines the quadrant of operation

$$T_p = 1 \rightarrow v_a = \pm V_s$$

$$T_p = 0 \rightarrow v_a = 0$$

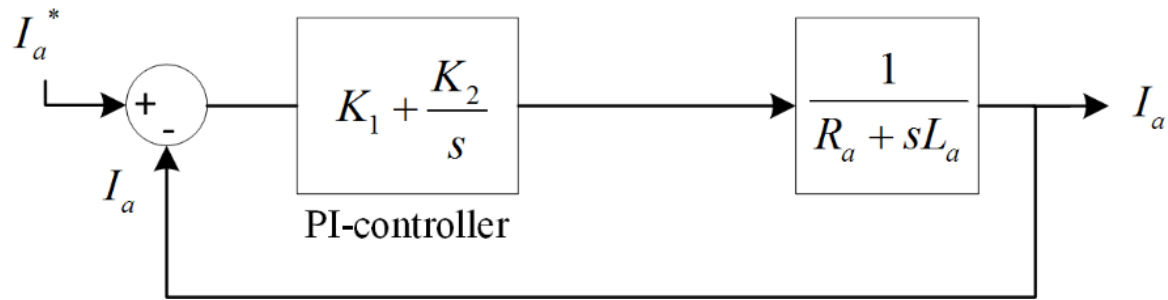
- PWM current controller

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \xrightarrow{\text{Laplace}} V_a = (R_a + sL_a) I_a + E_a$$



$$I_a = T_1 I_a^* + T_2 E_a$$

$$T_1: E_a = 0$$



$$T_1 = \frac{G_1}{1 + G_1 H_1};$$

$$G_1 = \frac{K_1 s + K_2}{(R_a + sL_a)s}; \quad H_1 = 1$$

$$T_1 = \frac{K_1}{L_a} \frac{s + \frac{K_2}{K_1}}{\underbrace{s^2 + \left(\frac{K_1 + R_a}{L_a} \right) s + \frac{K_2}{L_a}}_{s^2 + 2\xi\omega_n s + \omega_n^2}};$$

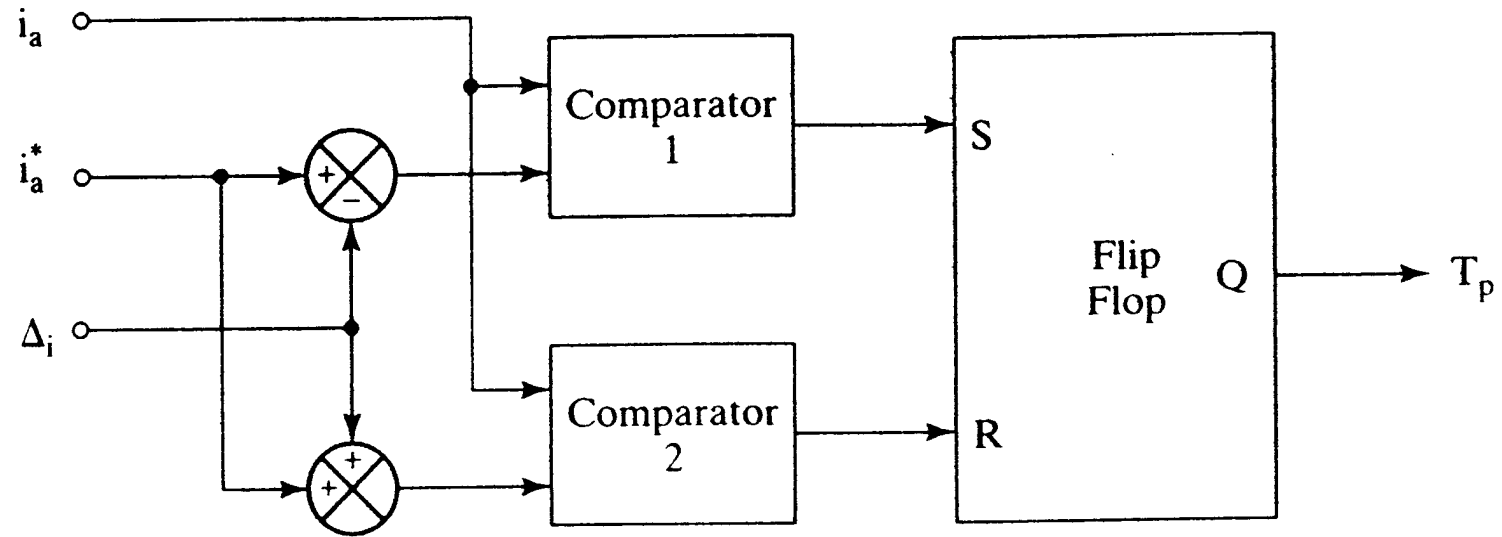
ξ : Damping ratio, ω_n : Natural frequency

$\xi < 1 \rightarrow$ Under-damped

$\xi = 1 \rightarrow$ Critically damped

$\xi > 1 \rightarrow$ Over-damped

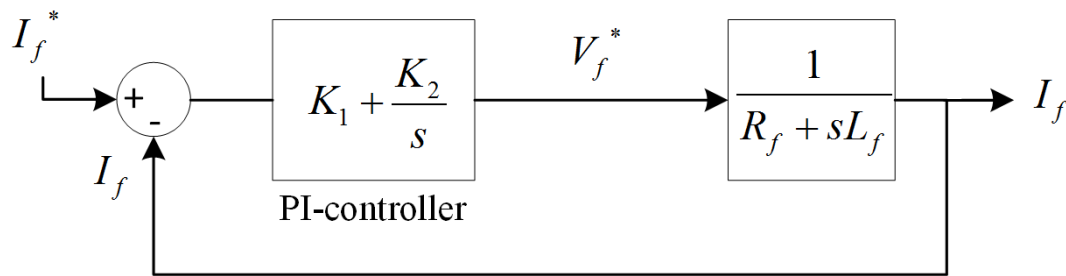
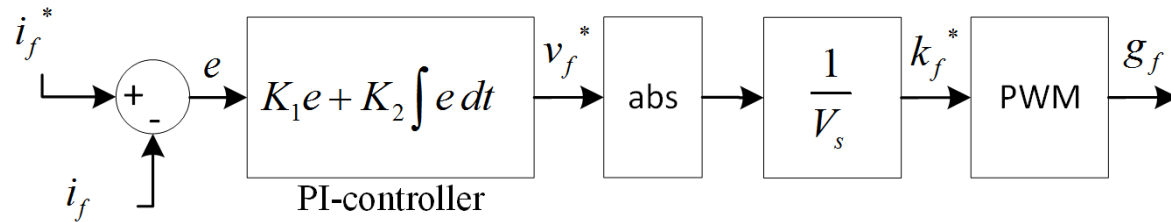
- Hysteresis current controller



- Flux Controller
 - PWM flux controller
 - Hysteresis flux controller

- PWM flux controller

$$v_f = R_f i_f + L_f \frac{di_f}{dt} \xrightarrow{\text{Laplace}} V_f = (R_f + sL_f) I_f$$



$$T = \frac{G}{1 + GH};$$

$$G = \frac{K_1 s + K_2}{(R_f + sL_f)s}; \quad H = 1$$

$$T = \frac{K_1}{L_f} \frac{s + \frac{K_2}{K_1}}{s^2 + \left(\frac{K_1 + R_f}{L_f} \right) s + \frac{K_2}{L_f}};$$

$\underbrace{\hspace{15em}}_{s^2 + 2\xi\omega_n s + \omega_n^2}$